This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

SGB
P88
CUTTER
OVERSIZE

WENDT

PERSPECTIVE

INCLUDING

THE PROJECTION OF SHADOWS AND REFLECTIONS

SPECIALLY PREPARED FOR ART STUDENTS

BY

ROBERT PRATT

Author of "Sciography"

Head-Master School of Art, Barrow-in-Furness

LONGMANS, GREEN, AND CO.

39 PATERNOSTER ROW, LONDON

NEW YORK AND BOMBAY

1901

Library
of the
University of Wisconsin

PERSPECTIVE

PERSPECTIVE

INCLUDING

THE PROJECTION OF SHADOWS AND REFLECTIONS

SPECIALLY PREPARED FOR ART STUDENTS

BY

ROBERT PRATT

AUTHOR OF "SCIOGRAPHY"

HEAD-MASTER SCHOOL OF ART, BARROW-IN-FURNESS

LONGMANS, GREEN, AND CO.

39 PATERNOSTER ROW, LONDON

NEW YORK AND BOMBAY

1901

All rights reserved

73624
T 1 .P88
SGB
P88

PREFACE

If an apology is wanted for this attempt to furnish art students with *still another* work on Perspective, the following facts may be deemed sufficient.

For the requirements of students in what was formerly the Elementary Stage of Perspective, exhaustive enough treatises were to be had, wherein the usual geometrical models and their combinations and amplifications were set forth in accordance with the Syllabus of the Board of Education. As was to be expected, however, these confined themselves to the *putting into* Perspective simple solids and objects of somewhat humble pretensions, from an architectural point of view, as dog-kennels, cradles, etc., in close association with a horizontal plane.

The more complete solution of the problem involved in the determination of shadows and reflections, under varying conditions of lighting, being confined to the Advanced Stage of the subject.

And here, again, books have been prepared, and commanded respect for their treatment of the subject from a geometrical point of view. The latter class-books are, however, in price somewhat beyond the reach of the ordinary student, and are only available when the elements of the subject have been otherwise acquired. The author has consequently been induced to prepare a series of lessons, going over the course as laid out in the new and interesting Syllabus of 1901, whereby it may be possible, within the period of an ordinary school session, to attain such an all-round acquaintance with the subject as shall be sufficient to induce the student to continue, by observation and drawing-board use, still further advancement in this valuable study.

Linear Perspective or Radial Projection, as the representing on a plane surface objects as they appear to the eye, is, from a science standpoint, properly included in the syllabus of "Plane and Solid Geometry."

It forms, however, an essential part in all practical art training, and, consequently, the enlargement of the art syllabus, and the increased importance given to the subject recently, must be received by art teachers with satisfaction.

Of what consequence, it may be asked, is it to an artist, this geometrical study of the subject, if he copy faithfully what he sees? To this the reply may be given—quoting from Leslie's "Handbook to Young Painters"—"that it is of the greatest consequence if it enables him to see better what he copies."

If after a course of lessons, wherein the principles and practice of Perspective, to the inclusion of the projection of shadows, have been fairly well mastered, we may, with confidence, critically examine a group of geometrical models drawn and *shaded*—under certain prescribed conditions—surely it is not too much to assume that the same training, or an extension of it on

the same lines, may enable one to analyze—without undue presumption—the treatment of, say, an interior where the scheme of lighting is evident.

We claim here, of course, only the ability to measure the truthfulness of the work as regards the *masses* of shadow or shade, and the correct delineation of the *reflected images*, where such may be included.

As regards the treatment of surface as seen under reflected lights, or half lights and half shades, etc., this vocabulary does not apply to the modest work aimed at here. Again quoting Leslie, "Colour and chiaroscuro have so entire a dependence on each other that they can never be treated separately." The scale, and the distances respectively of the eye from the picture plane and from the horizontal or ground plane, vary throughout the work; students should not be trained to assume a ten-feet (more or less) distance from the picture plane and a five-feet height of eye, with models built to suit. They should, also, in *all* cases, work out the problems to twice the scale given in the diagram.

Use has been made pretty freely of what has been termed the "direct method" of obtaining a perspective representation, viz. the combination of plan and elevation. This treatment is, however, neither artistic nor is it expeditious. These illustrations, together with the isometric views, are merely adopted to teach, in as graphic a manner as possible, the truths of the principles underlying the various rules laid down.

CONTENTS

PLATE		PAGE
	Syllabus of Art Examination	1
I.	Theory of Vanishing Points	2
II.	Circles and Curved Figures	6
III.	Horizontal Rectangle with its Edges inclined to the P.P.	8
IV.	A Square of 3″ Edge in Various Horizontal Positions, and in Vertical Planes Parallel to and inclined to the P.P.	10
V.	A Square Frame lying in a Horizontal Plane	12
VI.	A Square Pier in Two Positions	14
VII.	Square Pyramid with a Triangular Face in a Horizontal Plane	16
VIII.	A Cylinder with Axis Horizontal and a Cone lying on a Horizontal Plane	18
IX.	Planes Perpendicular to the P.P.	20
X.	A Rectangular Box with Open Lid	22
XI.	Planes in Various Positions	24
XII.	Pyramid on a Plane inclined to the G.P. and Perpendicular to the P.P.	26
XIII.	Cube resting on an Ascending Plane	28
XIV.	Isometric Illustration of the Above	30
XV.	A Writing-desk on an Ascending Plane	32
XVI.	Hexagonal Pyramid with Base on an Oblique Plane	34
XVII.	A Cube on an Oblique Plane with its Edges inclined to its H.T.	36
XVIII.	Projection of Shadows—Parallel Rays—Sun in the Plane of the Picture	38
XIX.	Shadow of Oblique Square when Sun is behind Spectator	40
XX.	Square Frame resting against a Prism—Sun behind the Spectator	42
XXI.	Shadow of Square Frame on Pyramid	44
XXII.	Shadows of Cylinder and Cone on Ground Plane—Sun in front of Spectator	46
XXIII.	Shadow of Prism on Cylinder—Sun behind Spectator	48
XXIV.	Portion of a Roof with Chimney Stalk coming through, shown in Perspective, to project Shadows when the Sun is in front of the Spectator (behind the P.P.) on the Left, Altitude 45°. Rays lying in Vertical Planes inclined 30° to the P.P.	50
XXV.	Radial Projection of Shadows—Source of Light a Point	52
XXVI.	The Perspective of a Hooded Doorway and the Position of L., an Artificial Light, being given to determine the Shadows	54

PLATE		PAGE
XXVII.	Reflections in Vertical Mirrors	56
XXVIII.	Reflections in a Horizontal Plane	58
XXIX.	Cube lying on an Oblique Plane with Reflection in a Vertical Mirror	60
XXX.	Reflection of Frame in Vertical Mirror	62
XXXI.	Gateway with Sloping Bank behind—Sun behind Spectator	64
XXXII.	Footbridge over a Stream—Sun in Plane of Picture	66
XXXIII.	Miscellaneous Problems	68
XXXIV.	Problems from Examination Paper (April, 1901)	70
XXXV.	Problems from Examination Paper (April, 1901)	72
XXXVI.	Problems from Examination Paper (April, 1901)	74

SYLLABUS OF ART EXAMINATION.

(The Examination will last two hours.)

PERSPECTIVE.

THE candidate will be expected to show—

(A.) Skill in using instruments and working out one or two problems accurately, and

(B.) Evidence of ability in the ready application of the rules of perspective to the representation of objects, views of buildings, landscapes, etc., by freehand sketches in pencil, ink, or water-colour. (In the last case, one colour only is to be used, such as sepia, lamp-black, or neutral tint.)

There will be first and second class passes. A second-class success will be accepted for the Elementary Drawing Certificate, and a first-class success for the Art Class Teachers' Certificate.

The number of marks awarded will be in proportion to the varying degrees of difficulty presented by the problems.

Candidates must qualify in (A.) and (B.).

(A.) Representing in perspective from plan and elevation, or from specification, simple solids or objects of plane or curved surfaces having one line or surface on, or parallel to, the ground-plane.

Drawing and measuring lines inclined to the horizontal, and contained in vertical planes inclined to the picture-plane.

Drawing figures or solids in perspective, some of whose leading constructive lines are horizontal, and the others contained in vertical planes at right angles to the horizontal lines, *e.g.* a cube with one edge horizontal, and one face making a given angle with the ground.

Drawing solids having plane or curved surfaces in oblique positions, and all their constructive lines inclined to the ground, such representations being limited to problems which can be solved by the treatment of an oblique plane and perpendiculars thereto.

Drawing reflections of solids in plane mirrors, horizontal or vertical.

Drawing shadows of lines, surfaces and solids, rectilineal or curved, upon any specified planes and on surfaces of single curvature, by natural or artificial light.

(B.) Finding and describing, from views given in perspective, the actual dimensions, position and other particulars respecting the objects represented under the conditions of any of the foregoing classes of subject (or in the case of shadows and reflections, ascertaining the position of the source of light, reflecting surface, etc.).

Indicating how change in position, say of the spectator, of the object, or of the source of light, etc., affects the representations of the given objects, etc.

Indicating effects of distance on the appearance of objects, shadows, etc.

Pointing out and correcting errors in perspective in respect of given subjects.

In the diagrams accompanying the paper of questions the following letters will represent the terms applied to the various points, lines, etc., used in working out the perspective scheme, viz. :—

H.L. Horizon Line.
C.V. Centre of Vision. The point on the picture-plane directly opposite the eye of the spectator.
E. Eye. The point showing the position of the eye of the spectator and its perpendicular distance from the picture-plane.
G.P. Ground Plane.
G.L. Ground Line. The intersection of the ground-plane with the picture-plane.
P.P. Picture Plane.
P.D. Point of Distance.
V.P. Vanishing Points of lines. (In the Plates **V.** only is used.)
V.L. Vanishing Lines of planes.
C.V.L. Centre of a Vanishing Line.
M.P. Measuring Point. (In the Plates **M.** only is used.)
S. The Sun.
V.P.S.R. Vanishing Point of the Sun's Rays. The point to which, when the sun is behind the spectator, the parallel lines representing the sun's rays appear to vanish.
V.L.P.S. Vanishing Line of the Plane of Shade. The **V.L.** of the plane passing through the **S.** or **V.P.S.R.**, and containing the line throwing the shadow.
L. Source of Artificial Light.
R.S. Reflecting Surface.

The candidate must show the points and lines by means of which the perspective or the geometrical result is arrived at, and also mark them respectively with the letters above given.

ADDITIONAL DEFINITIONS.

ORIGINAL LINE OR PLANE.—The line or plane belonging to the object which is to be drawn.

H.T.—Horizontal Trace. The point or line in which an original line or plane meets the ground-plane.

V.T.—The point or line in which an original line or plane meets the picture-plane; in the case of a plane this line is more generally termed the Picture Line (**P.L.**).

VANISHING PARALLEL.—In the case of lines—is a line drawn through the eye parallel to any original line, and which, when the line is inclined to the **P.P.**, will meet and determine the **V.P.** for the line. In the case of planes—it is a plane which, when it meets the **P.P.**, will determine the **V.L.** for the plane.

LINE OF DIRECTION is a line drawn through **E.**, the projection of the eye on the **P.P.**, parallel to the **V.L.** of the plane which is being treated in perspective.

CENTRE VISUAL RAY.—A line from the eye perpendicular to the **P.P.**, which determines the **C.V.**

FIELD OF VISION.—The space within which objects are assumed to be clearly visible. This is usually taken at as **30°**, a maximum, round the centre visual ray as an axis, and consequently a circle on the **P.P.** of this size should contain all perspective representations.

PLATE I.

LINEAR PERSPECTIVE is a geometrical treatment of the appearance of objects, when the relative positions of the spectator, the material under observation, and the transparent plane interposed between the spectator and this material (usually termed the Picture Plane) are specified.

Without any attempt at laying down the laws which are recognized as governing the Science of Optics, or any physiological analysis of the marvels of the human eye, we can assert that the *mental image* of any object within our vision, and our realization of its shape, surface, texture, and colour, is the result of plurality of vision. Both eyes, in general, have equal share in this transmission of visual phenomena; stereoscopic slides, with their interesting results, being constructed with this fact in view.

It is with geometrical rules, however, that we have here to concern ourselves. We have, then, as in Fig. 1, Plate I., the problem resolving itself into this: Given plan and elevation of a transparent plane—hereafter called the **P.P.** (Picture Plane)—which is vertical (though not necessarily so, as we shall see later on), reaching from the Ground Plane (**G.P.**) upwards indefinitely, the position of the eye (**E.**) (of necessity a point in geometrical perspective) and the projections of the material the perspective representation of which is to be obtained—in this case a square and an equilateral triangle—then, as light proceeds in straight lines from all points in the object to the eye, we need only obtain the intersection, with the **P.P.** of the various lines which join the corners of the figures and **E.** The original figures being rectilinear, we have then to join those points in the **P.P.** so determined, to obtain the outline termed the perspective representation of the original figures—the square and triangle.

The problem in detail is as follows:—

A square ABCD of $2\frac{1}{2}$" edge lies on a horizontal plane—here called the ground-plane—which is below the level of the eye; the nearest edge AB is parallel to and 1" from the P.P. AF, the continuation of AB to the left, forms the front edge of an equilateral triangle of 2" edge. Assuming also that E. is $2\frac{1}{2}$" above this horizontal plane and $5\frac{1}{2}$" from the P.P., required the intersection of the straight lines from AB, etc., to E. with this P.P. In plan by this means we get the horizontal, and in elevation the vertical, *displacements* which, combined, result in the shaded figures, ABCD representing the square and AFG the triangle. In combining, the **P.P.** in elevation is assumed to be turned round on the horizontal line—the line opposite **E.** I have already stated that this so-called "direct method" is cumbrous; and it now remains to determine the more artistic process, involving the use of Vanishing and Measuring points.

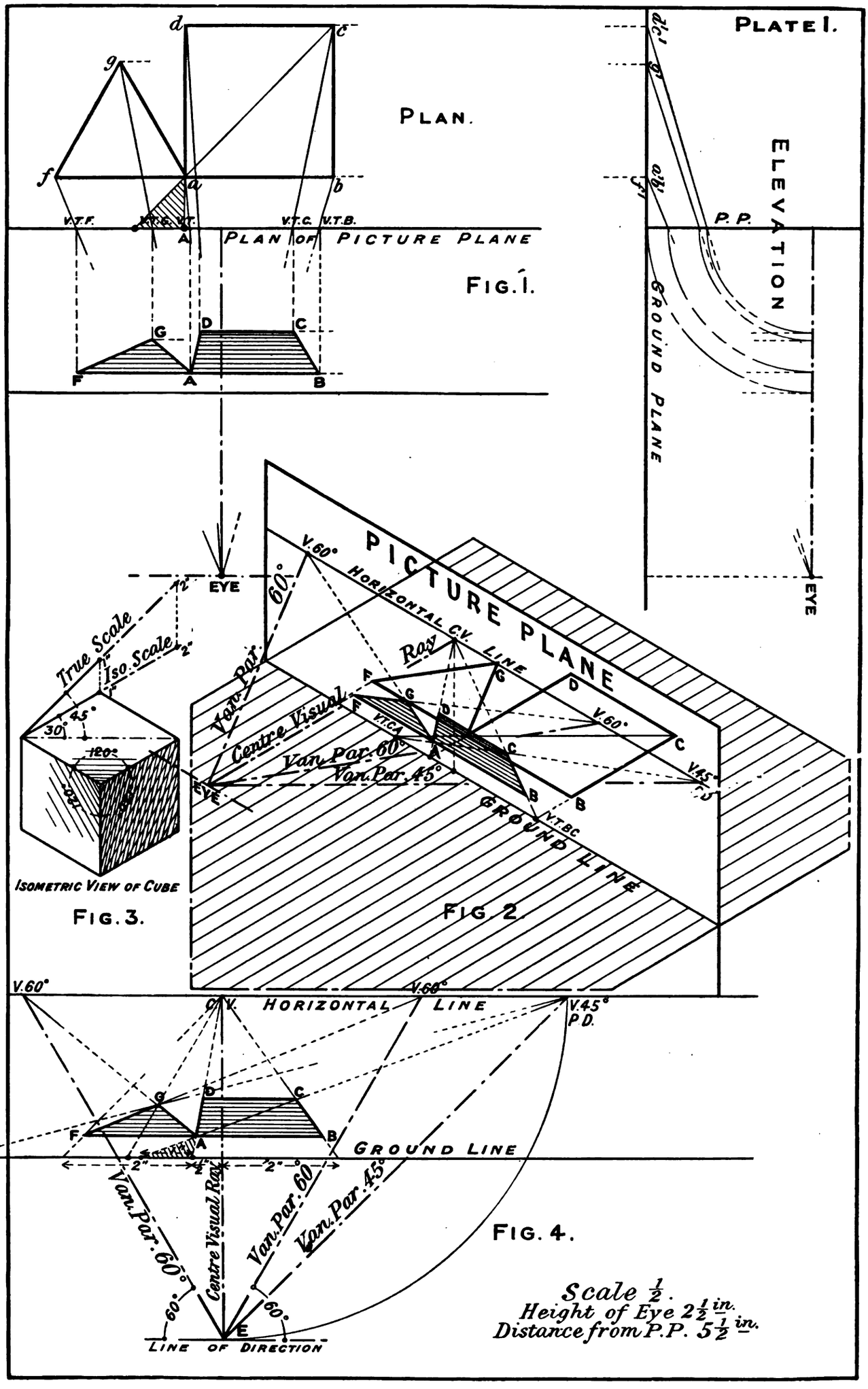
As is evident to any one with but an elementary knowledge of descriptive geometry, we have in the above merely determined the section of a pyramid of which the original figure is the base and **E.** the vertex; and it is only when the cutting plane is parallel to that of the pyramid base that the resultant section is similar to the base. We find in the perspective that DC, though parallel to, is shorter than AB, and hence AD and BC converge; produced, the point they converge to is found to be directly opposite **E.**

To explain, we turn to Fig. 2, which is an isometric projection of the group, set off in accordance with an isometric scale. For those unfamiliar with this system of projection, the view of the cube of 1" edge in Fig. 3 may be sufficient to show that when the three adjacent faces of a solid right angle (the angle of a cube) are equally inclined to the plane of projection, the face angles each measure **120°**; and the *projected* lengths of the edges—or the isometric scale—is determined from the true lengths on the **45°** line projected down to the **30°** line.

On this diagram, Fig. 2, by means of the **30°** set square and an isometric scale, the relative positions of the original figures on the ground-plane behind and the eye (**E.**) in front of the **P.P.** are shown, together with the perspective outlines as derived from Fig. 1 (Plan and Elevation). From **E.** a line termed the Centre Visual Ray is drawn perpendicular to **P.P.** (and therefore parallel to AD and BC), meeting it in **C.V.**, the centre of the "Field of Vision." The perspective representations of edges BC and AD, produced, meet in **C.V.**; that is, producing CB forward to meet the **G.L.** in **V.T.** BC, then **V.T.** BC joined to **C.V.** is the vertical trace, or intersection with the **P.P.** of a plane which passes through BC and **E.**, and so, in accordance with the definition of a plane, all lines passing from BC (or its continuation) to **E.** must intersect the **P.P.** in this **V.T.**, and similarly with edge AD. That is, *the vanishing*

NOTE.—It is not desirable that the student draw these isometric illustrations, though the teacher may find it of advantage to prepare large diagrams in this manner for class instruction.

PLATE I.



Scale $\frac{1}{2}$.
Height of Eye $2\frac{1}{2}$ in.
Distance from P.P. $5\frac{1}{2}$ in.

PLATE I.—*Continued.*

point for all lines perpendicular to the plane of the picture is the Centre of Vision. Next, to determine the portions which will represent AD and BC and the distance of A from the **G.L.**—or its distance *within the picture*—diagonal CA is produced in like manner to cut the **G.L.** in **V.T.** CA, and through **E.** a line is drawn parallel to AC, meeting the **H.L.** in **V.P. 45°**. Join **V.T.** AC and **V.P. 45°**, and again this will be the perspective representation of a line infinite in length which will contain AC.

In like manner, by drawing through **E.** lines parallel to FG and AG respectively, to meet the **H.L.** at angles of **60°**, *i.e.* setting off at **E.** lines at **60°** with the "Line of Direction" (parallel to **H.L.**), we obtain the **V.P.'s** for FG and AG.

To sum up, we have the following rules :—

The perspective representation of an original line, or set of parallel lines, inclined to the P.P., will converge or vanish to a point in the P.P.

To obtain this point—the V.P.—draw, parallel to the original line, a line through E. to meet the P.P.

All horizontal lines will have their V.P.'s in the H.L.

All lines perpendicular to the P.P. will have C.V. for their V.P.

Lines parallel to the P.P. will have no V.P.

Their perspective representations consequently will be lines parallel to the original lines.

To measure distances on lines which are perpendicular to the P.P., *i.e.* which vanish to the C.V., we use the V.P.'s for horizontal lines which make 45° with the P.P. These points, M.P.'s for lines vanishing to the C.V., specially called Points of Distance (P.D.), are best obtained by setting off on the H.L. on each side of C.V. distances equal to that of E. from C.V.

The foregoing explanation of the processes involved in obtaining a perspective representation may seem unduly prolonged, but it seems better and more logical thus to introduce a student to the principles underlying the art of perspective delineation at the outset, than to insist on a mechanical *putting into perspective* of all sorts and conditions of objects for months, and then doling out theory as a separate diet.

The perspective diagram (Fig. 4) should now be understood. Corner A is $\frac{1}{2}$" to the left of the centre and 1" from the **G.L.**—AB being parallel to the **G.L.** Set off $\frac{1}{2}$" to the left of the centre of **G.L.** and join to **C.V.**; this will contain edge AD.

Set off 2" to right of centre (AB being $2\frac{1}{2}$") and join to **C.V.**; this will contain BC. Set off $2\frac{1}{2}$" to left of centre and join to **C.V.**; this will contain F, because AF = 2". Now, because FAB is parallel to and 1" from the **G.L.**, we can measure this 1" distance along any of these three lines. For that purpose we use the **V.P.** for lines making **45°** with the **P.P.** or the **P.D.** Set off to left of **V.T.** AD 1" in the **G.L.** and join to the right-hand **P.D.**; by this means we *locate* not only A, but we also obtain the diagonal AC, and, because FAB is parallel to the **P.P.**, *i.e.* parallel to **G.L.**, it has consequently no **V.P.** We draw it parallel to **G.L.**, and complete the representation of the square by drawing through C another parallel CD.

To complete the triangle, the **V.P.'s** for edges FG and AG are obtained by drawing their vanishing parallels through **E.** to meet the **H.L.**

Corner G could have been obtained otherwise by a perpendicular through G to meet the **G.L.**, which will, of course, vanish in **C.V.**; and measuring its distance within the picture, *i.e.* its distance from **G.L.**, by the use of the **P.D.** Both methods are shown.

PLATE I.

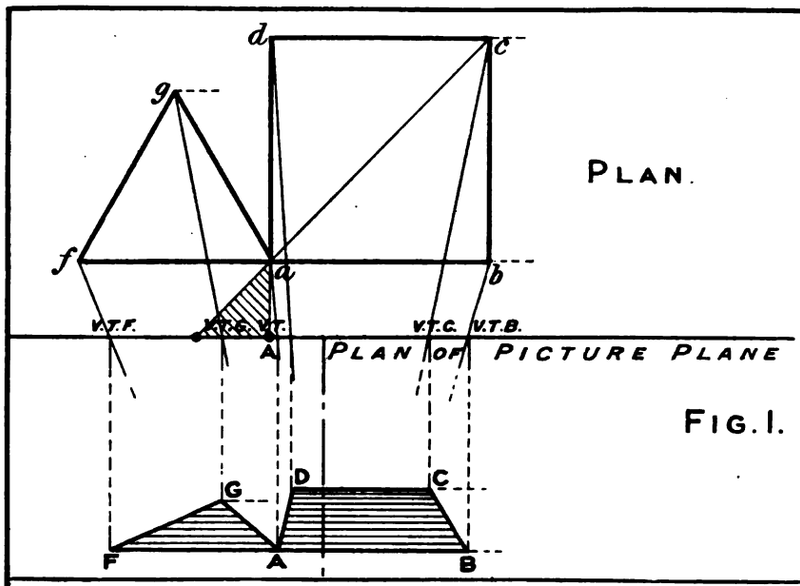
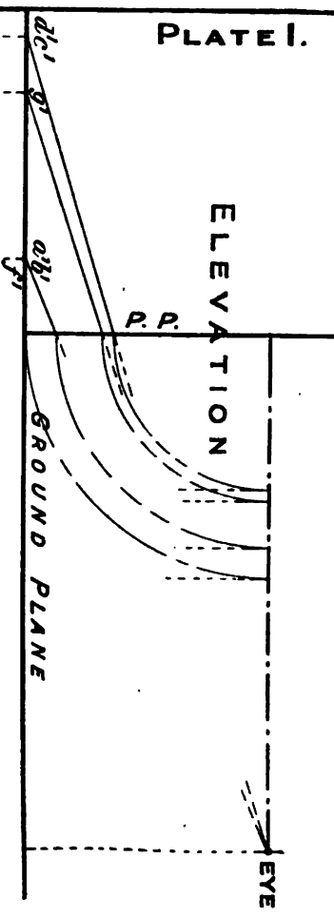


FIG. 1.



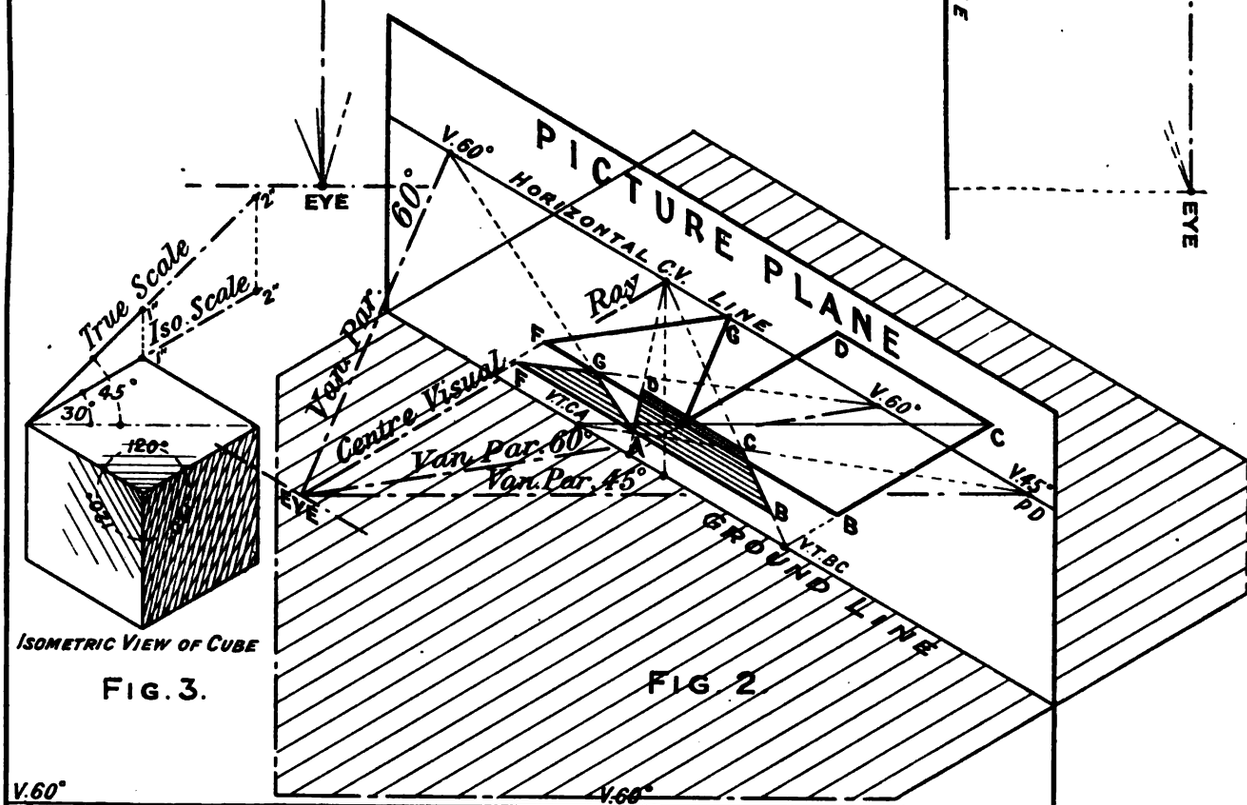


FIG. 2.

FIG. 3.

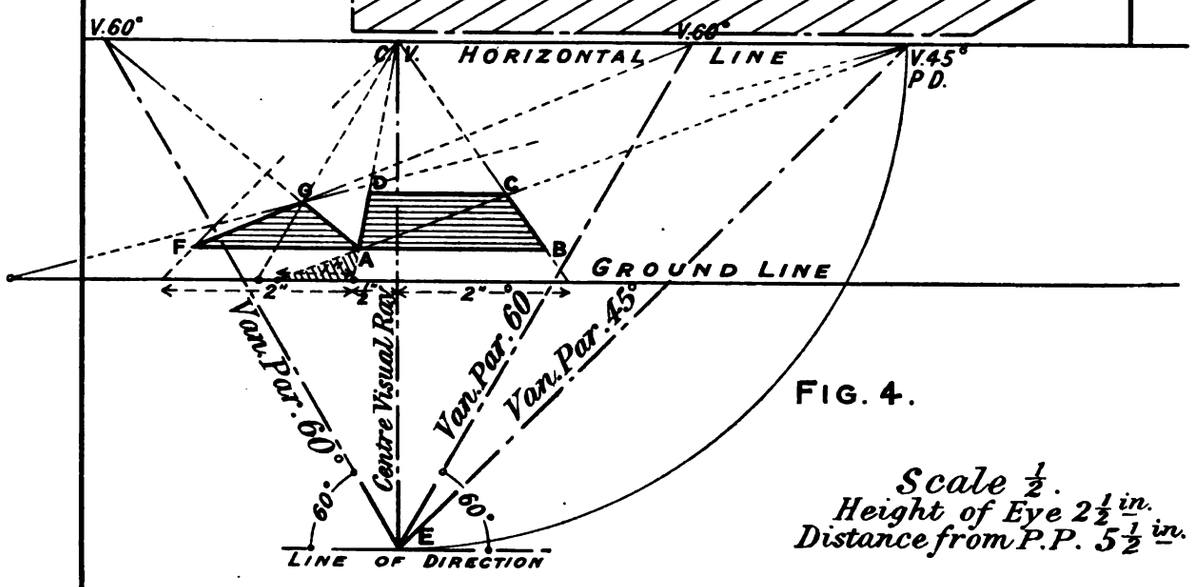


FIG. 4.

Scale $\frac{1}{2}$.
Height of Eye $2\frac{1}{2}$ in.
Distance from P.P. $5\frac{1}{2}$ in.

PLATE II.

CIRCLES AND CURVED FIGURES.

The representations of circles, and generally of curved figures, are obtained by enclosing them in the most suitable network of straight lines. In the case of a circle the lines of its enclosing square and diagonals are evidently the best.

Fig. 1 shows in plan and elevation a sphere directly in front but below the level of the eye. The visible portion of this sphere will be obtained by enveloping it in a cone of which **E.** is the vertex. The contour of the visible portion—less than half—will be the circular tangent of cone and sphere, AB; and the section of this circumscribing cone by the **P.P.** will be an ellipse.

The perspective representation, therefore, of a circle when its plane is not parallel to that of the **P.P.** will be an ellipse: it will, of course, be a circle when its plane is parallel to the **P.P.**

Fig. 2 shows the representation of a circle 2″ diameter in three different positions:—A is in a horizontal or ground plane (G.P.) 3″ below the eye, its centre being $1\frac{1}{4}$″ to the left and $1\frac{3}{4}$″ within the picture. B touches the same plane, but stands vertical, its plane perpendicular to P.P., and its point of contact with G.P. $1\frac{1}{4}$″ to right of centre and $1\frac{3}{4}$″ within the picture. C stands on the same G.P., is vertical, its plane being parallel to that of the P.P., and its point of contact with G.P. being directly in front, and $3\frac{1}{2}$″ from the G.L.

For explanatory reasons the group is shown in plan and elevation; the perspective representation has, however, been obtained by the use of **V.** and **M.** points.

To find centre A, set off **V.T.** 13 $1\frac{1}{4}$″ to left of centre on **G.L.**, join to **C.V.** and measure along this $1\frac{3}{4}$″ by using **P.D.** (either). This will determine centre A and one diagonal of enclosing square.

On each side **V.T.** 13 mark off radius of circle and join to **C.V.**, and then complete enclosing square. The parallel lines through points where circle is cut by diagonals are also obtained, and the ellipse—the resulting perspective—drawn neatly through the eight points thus identified.

For B set off on **G.L.** $1\frac{1}{4}$″ to right and join to **C.V.**; this will contain base edge or **H.T.** of upright square. Set off at same point a perpendicular to **G.L.**, which will represent the **V.T.**, or, as it is termed in the Syllabus, the Picture Line; **P.L.** of the vertical plane containing circle B.

Measure on this **P.L.** the diameter of the circle and join to **C.V.** This the perspective of a horizontal line in the vertical plane of the circle will contain the upper edge of the enclosing square.

Measure along the **H.T.** $\frac{3}{4}$″ and beyond this 2″, and uprights through these will complete the square. The centre being found diagonals drawn and cut for points 5, 6, 7, 8, either by use of **C.V.** or **P.D.**; then complete, by freehand, the ellipse.

For circle C, which is parallel to **P.P.**, and therefore in perspective is a circle, what we seek for is position of its centre and radius, or bottom edge and length; for enclosing square measure directly in front of spectator, that is, from centre of **G.L.** to **C.V.**, $3\frac{1}{2}$″ by use of **P.D.**; a line through this parallel to **G.L.** will contain bottom edge of square for circle C, and its half-length is measured on either side by a line to **C.V.**

Fig. 3 shows the perspective of two curved plane figures, the plans of which are given in order to indicate their size and position with reference to the P.P. The perspective result may be followed out without any special detailed description. In order to economize work, students must decide which points in the curves it is of most importance to locate.

PLATE 2.

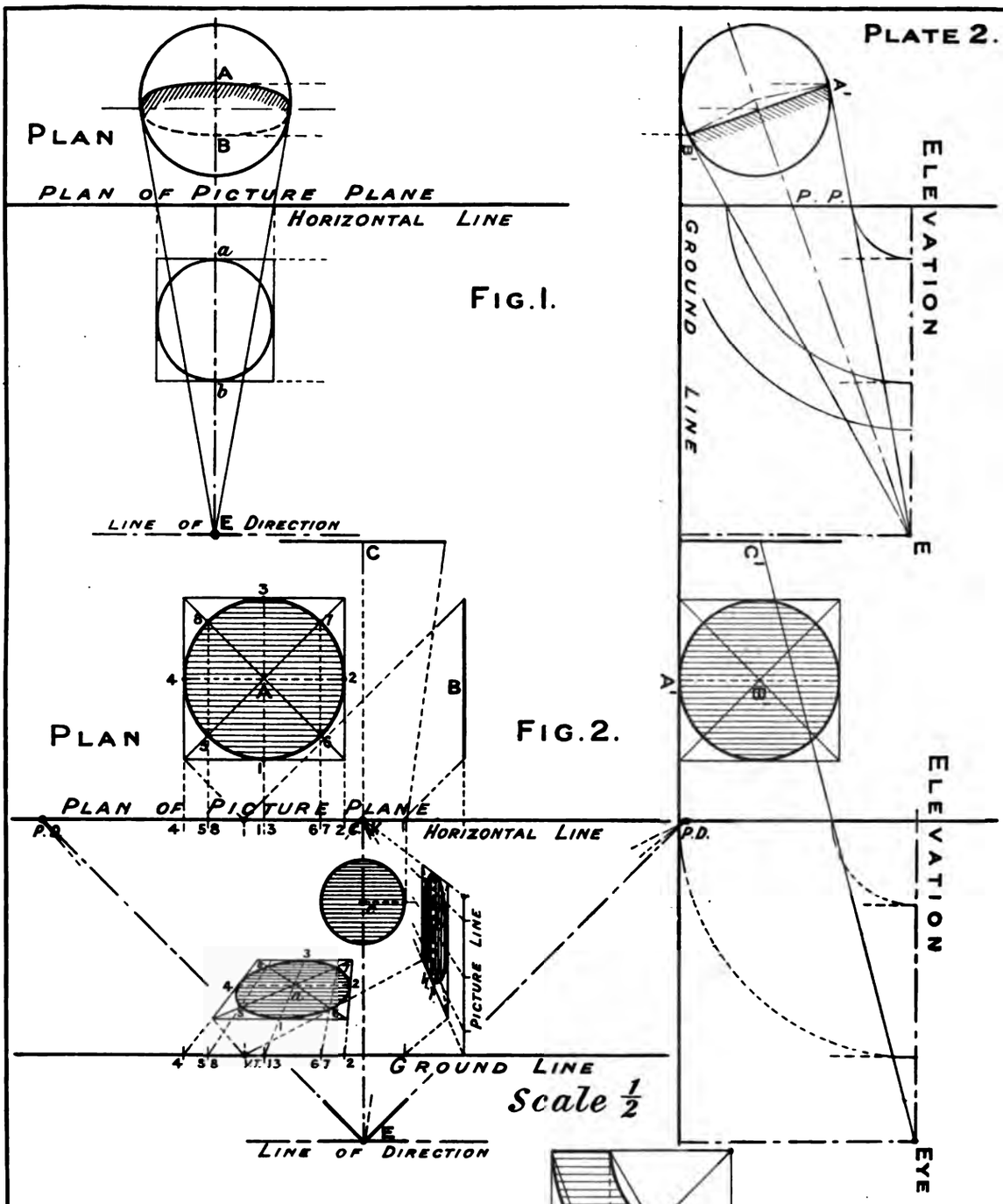


FIG. 1.

FIG. 2.

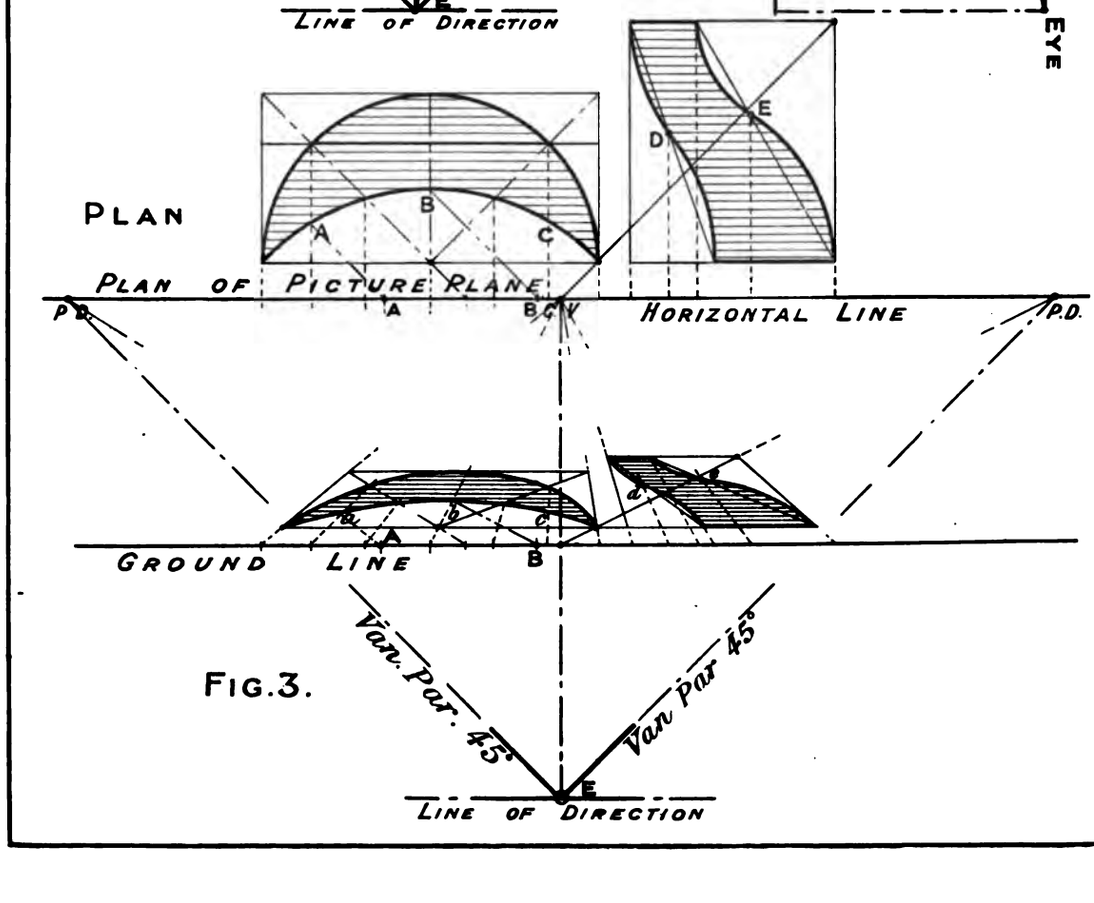


FIG. 3.

PLATE III.

HORIZONTAL RECTANGLE WITH ITS EDGES INCLINED TO THE P.P.

When the edges of a figure, though horizontal, are not perpendicular to the **P.P.**, and consequently do not vanish to the **C.V.**, but to some other point in the **H.L.**, their **V.P.'s** may be determined—as has been already explained with reference to the equilateral triangle in Plate I.—by drawing through **E.** a line parallel to the original line. This line—the Vanishing Parallel—will meet the **H.L.** when the original line is horizontal: will meet it above or below according as the original line ascends above or descends below the horizontal.

As regards measuring such a line, confining ourselves in the mean time to horizontal lines, the method associated with the horizontal rectangle shown in Plate III. will explain the theory and practice of Measuring Points.

ABCD is a rectangle $3\frac{1}{2}'' \times 2\frac{1}{2}''$ lying in a horizontal plane $3\frac{1}{2}''$ below the eye; its edges are equally inclined to the P.P. or G.L. As will be seen in plan, the nearest corner is directly in front and 1″ within the picture.

Produce BA forward to meet the **P.P.**, that is, find **V.T.** AB; and taking this as centre turning distances A and B round to **P.P.**, and joining to A and B, we obtain isosceles triangles with **V.T.** AB as a common vertex.

The angle at the vertex is **45°**, and consequently the base angles will be **67½°** each $\left(\frac{180° - 45°}{2}\right)$. So that to measure edge AB, which vanishes **45°** to the right, use **V.P.** for **67½°** on the left; and similarly with edge AD, vanishing **45°** to the left, use **V.P. 67½°** on the right.

To obtain and use these **M.P.'s**: having found **V.P.** for lines vanishing **45°** to the right by setting off at **E. 45°** with the Line of Direction, bisect the remaining angle, **135°**, thus obtaining the **V.P.** for lines vanishing **67½°** to the left; or—

Rule for obtaining Measuring Points for horizontal lines: From a V.P. set off on H.L. the length of its Vanishing Parallel, or V.P. to E. This will give its associated M.P.

When, as sometimes happens, a **V.P.** may be beyond the limits of the paper, and still in use, obtain its **M.P.** by the method of bisecting the angle at **E.**

To use an M.P.—Through **M.P.** and the point on the line from which a distance is to be measured, draw a line to meet **G.L.** Set off from this the length required, and the return line to **M.P.** will cut off the perspective length.

As in the example here given, because A is 1″ directly in front, set off 1″ to left of centre of **G.L.** and join to **P.D.**, which being **V.P.** for **45°**, will give line containing AB. To measure AB find **M.P.** for AB—for lines vanishing **45°** to the right—and through it and A draw a line to meet **G.L.** From this set off $3\frac{1}{2}''$, and drawing back to **M.P.** we cut off the length AB required. And similarly for AC, which is $2\frac{1}{2}''$. The resulting figure may also be checked by determining the **V.P.** for its diagonal AC.

PLATE 3.

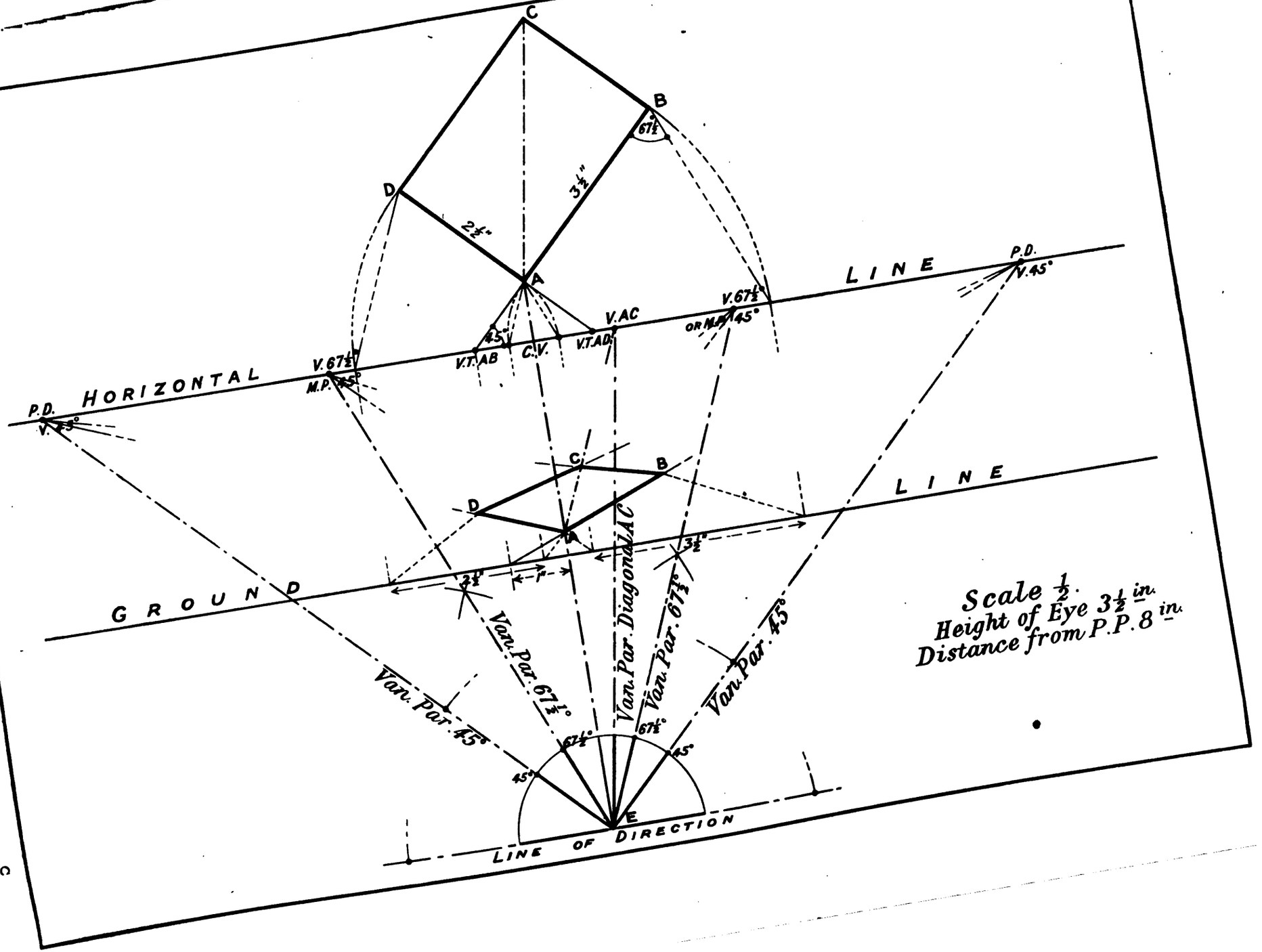

C
B
D
A
67½°
3½"
2½"
HORIZONTAL
LINE
P.D.
V.45°
V.67½°
V.AC
45°
V.T.AB
C.V.
V.T.AD
M.P.
GROUND
Van. Par. 45°
Van. Par. 67½°
Van. Par. Diagonal AC
Van. Par. 67½°
Van. Par. 45°
Scale ½.
Height of Eye 3½ in.
Distance from P.P. 8 in.
LINE OF DIRECTION
E

PLATE IV.

A SQUARE OF 3″ EDGE IN VARIOUS HORIZONTAL POSITIONS, AND IN VERTICAL PLANES PARALLEL TO AND INCLINED TO THE P.P.

All the lines of any figure in perspective must be measured—as has been already shown in Plate III.—on the **P.L.** of the plane containing the figure, or on that of *any* plane containing the line.

Squares A and B lie in a horizontal plane—here called the ground-plane —which is 4″ below E.

A has its edges parallel and perpendicular to the P.P. and its nearest corner 1″ to the left and 1″ within.

When a horizontal plane is called the ground-plane, its **P.L.** is specially called the **G.L.** Measurements on the **G.L.** determine the perspectives of squares A and B.

The nearest corner of B bisecting the back edge of A, and its edges being equally inclined to the **P.P.**, are measured by the use of the **M.P.'s** for **45°**.

Squares C and D, which are related to each other and the P.P. in a similar way to A and B, lie in a horizontal plane which is 3″ above E., nearest corner of C 1″ to the right and 2″ within the picture. We use the **P.L.** for this horizontal plane for measuring purposes in the same way as we use the **G.L.** for figures on the ground-plane.

Square E is parallel to the P.P., and with one edge on the G.P., near end of it $\frac{1}{2}$″ to right and 12″ within.

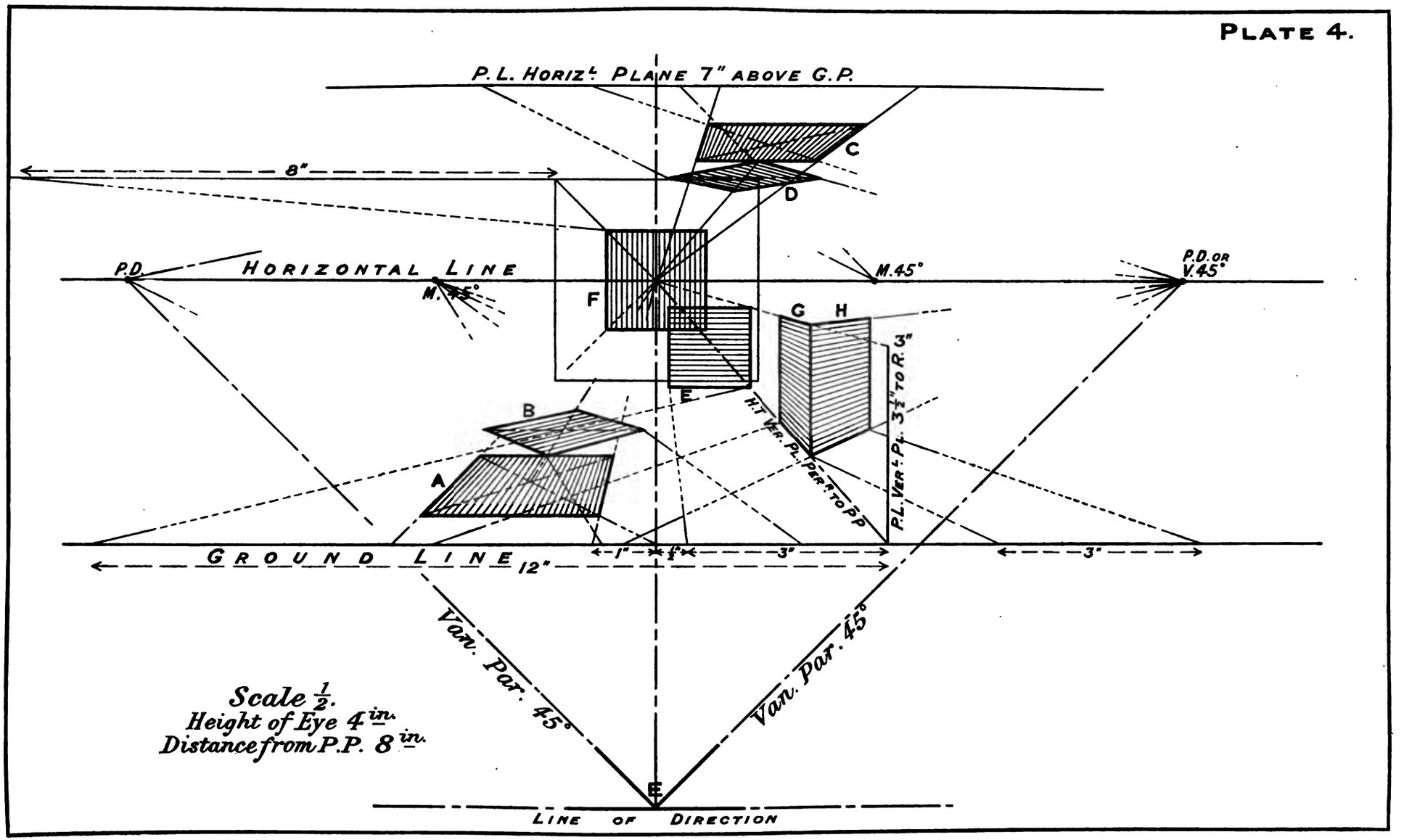
F is also parallel to and 8″ from the P.P., but is directly opposite E.

Square G lies in a plane perpendicular to the P.P., which is $3\frac{1}{2}$″ to the right, and therefore coinciding with right-hand edge of square E, near edge 4″ within the picture.

Hinged on this same edge is square H, its plane inclined 45° to the right.

The working out of these problems may be followed out without any further detailed description.

PLATE 4.



Scale $\frac{1}{2}$.
Height of Eye 4 in.
Distance from P.P. 8 in.

PLATE V.

A SQUARE FRAME LYING IN A HORIZONTAL PLANE.

A square frame $2\frac{1}{2}$″ edge, material $\frac{1}{2}$″ square, lies in a horizontal plane $2\frac{1}{2}$″ below the eye. Its edges vanish 30° to the right and 60° to the left respectively. Nearest corner of under face A, $\frac{1}{4}$″ to the left and $\frac{1}{2}$″ within the picture.

Vanishing parallels through **E.** will give the **V.P.'s** required, and with these we determine the corresponding **M.P.'s.**

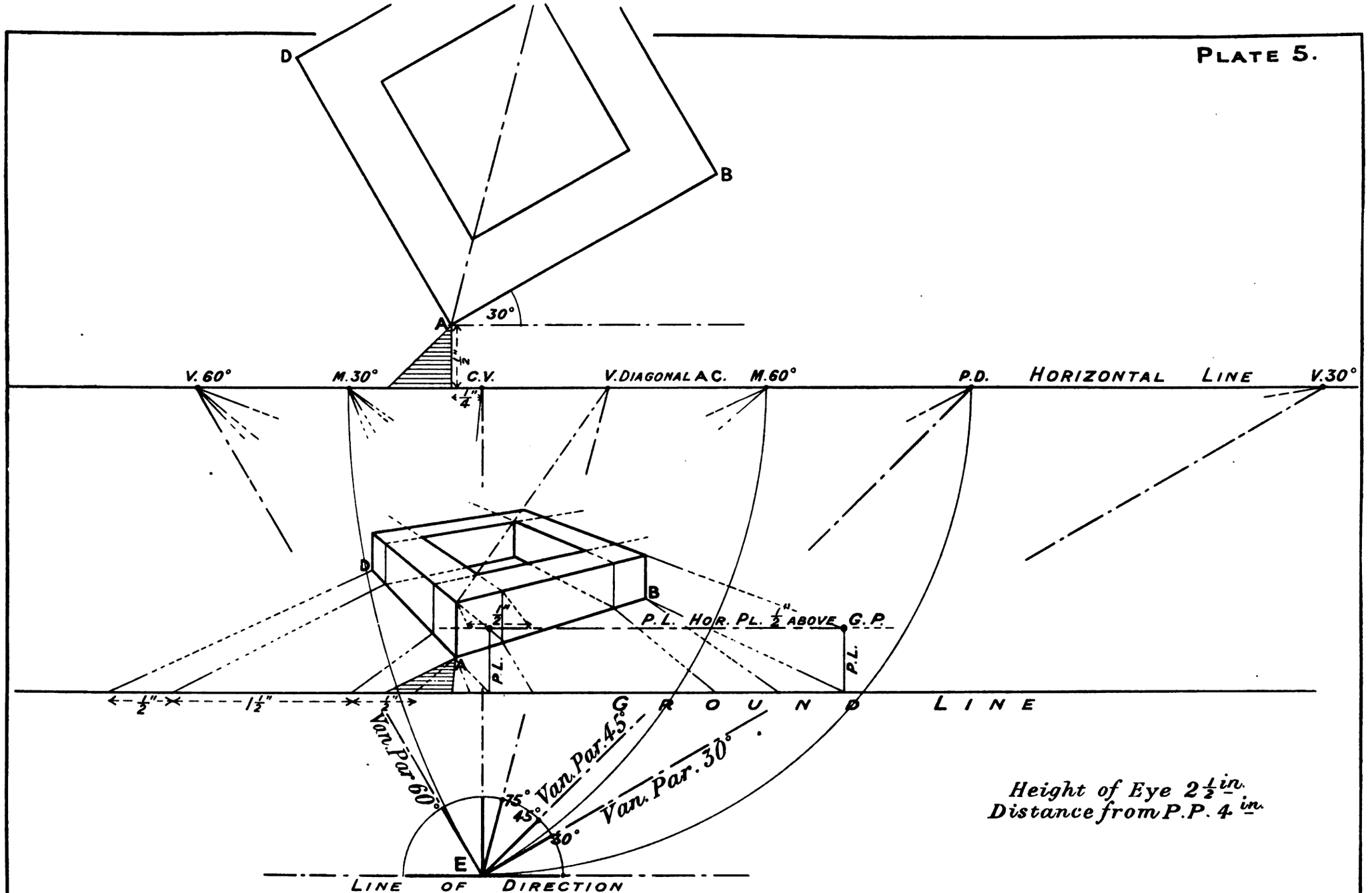
And, as will be obvious, the **M.P.** for lines vanishing **60°** will be the **V.P.** for lines vanishing also at **60°** in an opposite direction.

It must be remembered, however, that the **C.V.** and **P.D.** are used in order to locate the *start* point A. Having obtained the perspective of the under square face, the upright edges will next be drawn and measured. In order to do this—in an economical way—and remembering that an upright line may be measured by the use of any point in the **H.L.** as an **M.P.**, we had better produce any suitable base edge to meet the **G.L.**: that is, draw the **H.T.** of the vertical plane of the side of the square frame, producing it forward to meet **P.L.**, and erecting through this the **P.L.** of that face.

Both sides, through DA and CB, have been shown so treated. On these **P.L.'s**, when the thickness is set off and taken back to the **V.P.**, we obtain one edge of the top, and so the other edges of the square are obtained by the use of the appropriate **V.P.'s.** The material being square will show a horizontal thickness also of $\frac{1}{2}$″, which may be measured, as shown, first on the edges of the under square and so brought vertically up to the top, or the upper thicknesses may be measured directly on the **P.L.** of the top.

One diagonal AC will vanish **15°** to the right of the centre.

PLATE 5.



Height of Eye 2½ in.
Distance from P.P. 4 in.

PLATE VI.

A SQUARE PIER IN TWO POSITIONS.

A pier, square in horizontal section, is shown in elevation, and the problem is to draw its perspective representation in two positions.

1st Position.—When its faces are parallel and perpendicular to the P.P.

2nd Position.—When these faces are equally inclined to the P.P.

The pier rests on a horizontal plane which is here appropriately termed the ground-plane, being $4\frac{1}{2}'$ below the spectator's eye. Scale $\frac{1}{2}'' = 1'$ or $\frac{1}{24}$.

1st Position.—The nearest corner of the square plinth is 1′ to the left and 2′ within the picture. After the square base on the **G.P.** has been drawn, the vertical edges of the plinth, and also the corresponding edges of the pyramidal cap, are next drawn and measured—the square part of the cap coinciding in *plan* with the plinth.

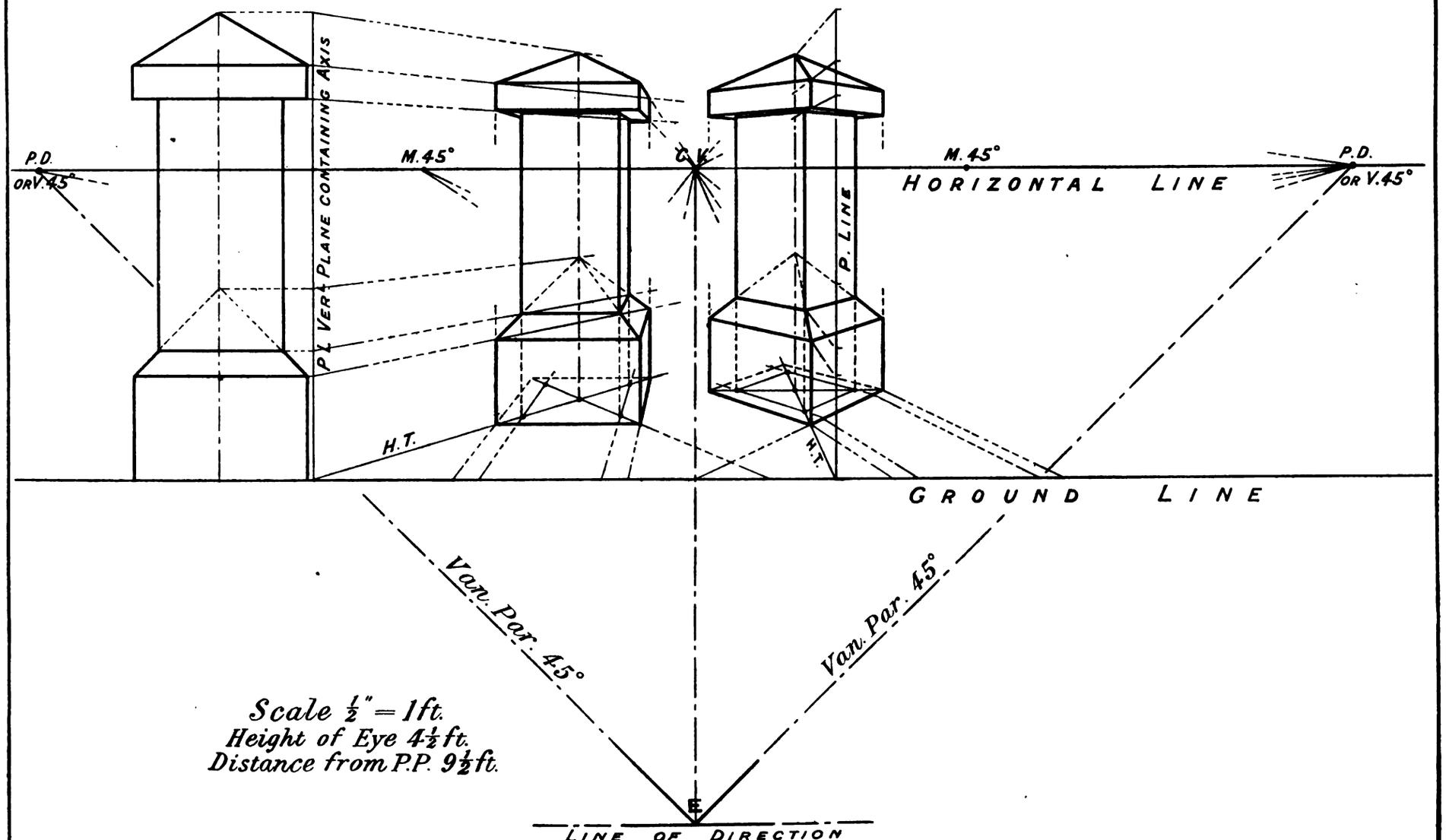
To measure all these as well as the vertex, we here adopt the **P.L.** of the vertical plane containing one diagonal of the base, and consequently also the axis and vertex. All heights, therefore, are measured on this line and taken to the **P.D.** on the right, which is the **V.P.** for all horizontal lines in this vertical plane.

2nd Position.—Nearest corner 2′ to the right and 2′ within the picture, faces equally inclined to the P.P.

The base edges here vanish at **45°**, and are measured by **M.P.'s 45°**. The diagonals are parallel and perpendicular to the **P.P.**, and so the heights are best measured by the use of the **P.L.** of the vertical plane containing the diagonals which vanish to **C.V.** In both problems notice that the vertical edges of the square die are projected from its plan, which has been drawn on the **G.P.** All heights are, however, measured on the same line. These examples illustrate what is commonly termed Parallel and Angular Perspective respectively.

In measuring heights, the student must remember that in all cases the height-line on which the various heights are set off must be the **P.L.** of a plane containing the line to be measured.

PLATE 6.



Scale ½" = 1ft.
Height of Eye 4½ ft.
Distance from P.P. 9½ ft.

PLATE VII.

SQUARE PYRAMID WITH A TRIANGULAR FACE IN A HORIZONTAL PLANE.

A pyramid, base $1\frac{3}{4}$" square, altitude $2\frac{1}{4}$", is to be drawn in perspective with one face in a horizontal plane, in two positions.

1st Position.—When the edge AB of the square base—the edge on the ground—is vanishing 60° to the left, and when the near corner A is $1\frac{1}{4}$" to the right and $1\frac{1}{4}$" within the picture.

To draw the perspective of the triangular under face—which is seen dotted in Plan—we may either determine the vertex **V.** by the bisecting line of the face, that is, from centre of edge AB a line vanishing **30°** to the right and its length measured, or by measuring angle $a°$, that between base edge AB and AV, and setting off at **E.** with the vanishing parallel for AB the same angle. This would give **V.P.** for edge **A.V.** The former method has been adopted.

Now, as the plane of the square base is not vertical, but hangs over, as it were, towards the vertex, we determine the position of corner D, and consequently edge AD in an *indirect* way. (There is a direct way we shall learn later on.)

That is, we determine *d* the *plan* of corner D on the ground, and set up above this a vertical line of the required length (obtained from the elevation). The shaded triangle in the perspective indicates that AD is really the hypothenuse of a right-angled triangle, of which A*d* is the base and *d*D is the altitude. A*d* being perpendicular to AB, and hence vanishing **30°** to the right; and D being thus obtained, the method adopted for finding edge DC is obvious.

2nd Position.—One edge AV of the triangular under face is parallel to and $\frac{3}{4}$" from the P.P., the pyramid vertex V. being directly in front.

After AV has been obtained, we next seek for the **V.P.** of the base edge AB by setting off at **E.** with the Line of Direction the angle $a°$ = BAV. We set it off with the Line of Direction because AB is parallel to the **P.P.**, and hence the Line of Direction represents its Vanishing Parallel. **V.P.** for AB being thus found, and AB measured by its appropriate **M.P.**, we next find corner D of upper base edge by the indirect method adopted in the first position. That is, a line in the **G.P.** perpendicular to AB is drawn and measured = A*d*, and through this the altitude line *d*D is erected.

As will be seen, the **V.P.** for A*d* is obtained at **E.** by a **90°** Vanishing Parallel to Vanishing Parallel AB; and, as this **V.P.** is beyond the paper, its **M.P.** had better be obtained by bisecting the angle at **E.**

PLATE 7.

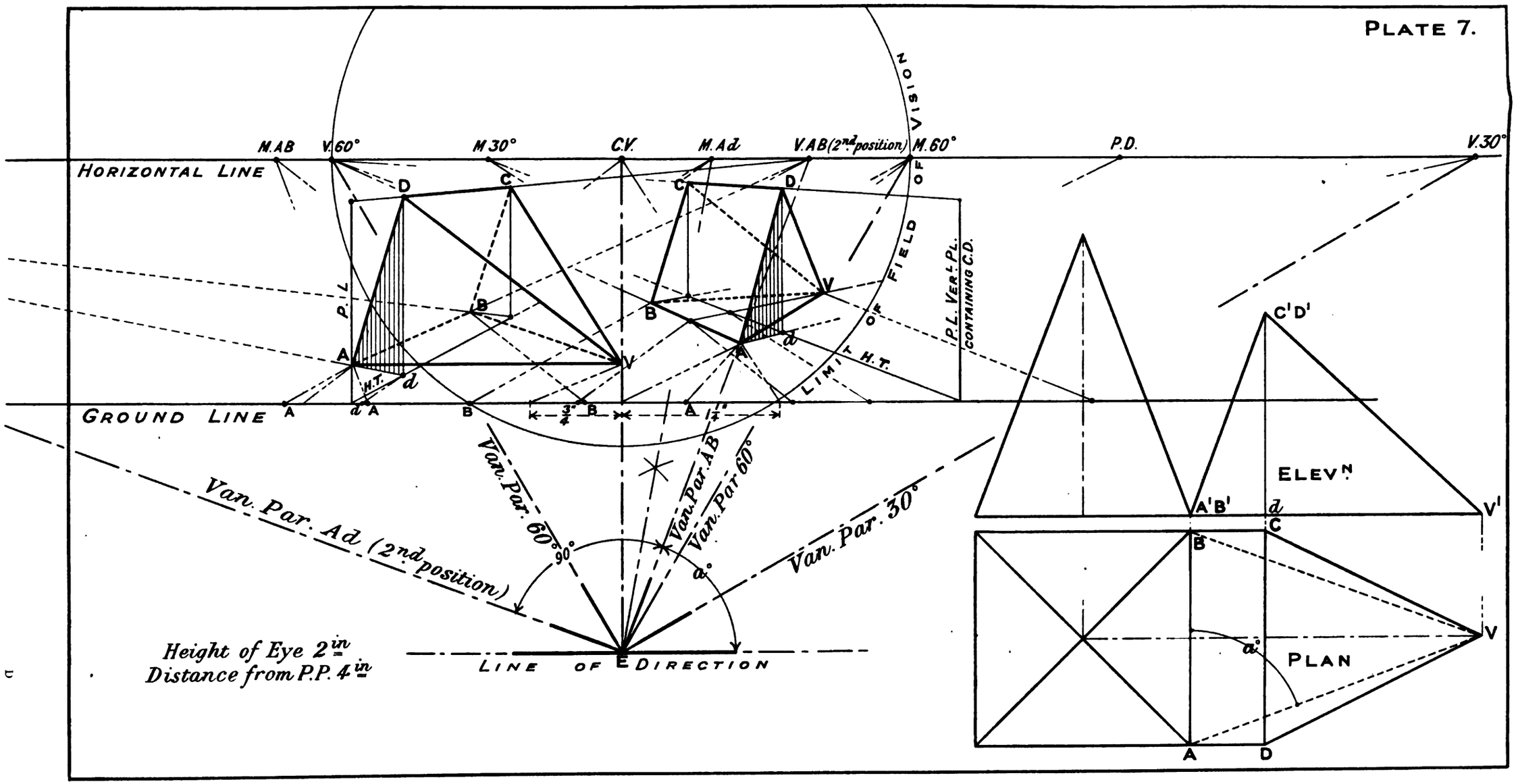

M.AB
V.60°
M.30°
C.V.
M.Ad
V.AB(2nd position)
M.60°
P.D.
V.30°
HORIZONTAL LINE
GROUND LINE
FIELD OF VISION
LIMIT OF H.T.
P.L. VERT. PL. CONTAINING C.D.
P.L.
H.T.
A
B
C
D
V
d
3/4"
1 1/4"
Van. Par. Ad (2nd position)
Van. Par. 60°
Van. Par. AB
Van. Par. 60°
Van. Par. 30°
90°
a°
LINE OF DIRECTION
Height of Eye 2in
Distance from P.P. 4in
ELEVN
PLAN
A'B'
C'D'
V'

PLATE VII.

SQUARE PYRAMID WITH A TRIANGULAR FACE IN A HORIZONTAL PLANE.

A pyramid, base $1\frac{3}{4}$" square, altitude $2\frac{1}{4}$", is to be drawn in perspective with one face in a horizontal plane, in two positions.

1st Position.—When the edge AB of the square base—the edge on the ground—is vanishing 60° to the left, and when the near corner A is $1\frac{1}{4}$" to the right and $1\frac{1}{4}$" within the picture.

To draw the perspective of the triangular under face—which is seen dotted in Plan—we may either determine the vertex **V.** by the bisecting line of the face, that is, from centre of edge AB a line vanishing **30°** to the right and its length measured, or by measuring angle $a°$, that between base edge AB and AV, and setting off at **E.** with the vanishing parallel for AB the same angle. This would give **V.P.** for edge **A.V.** The former method has been adopted.

Now, as the plane of the square base is not vertical, but hangs over, as it were, towards the vertex, we determine the position of corner D, and consequently edge AD in an *indirect* way. (There is a direct way we shall learn later on.)

That is, we determine *d* the *plan* of corner D on the ground, and set up above this a vertical line of the required length (obtained from the elevation). The shaded triangle in the perspective indicates that AD is really the hypothenuse of a right-angled triangle, of which A*d* is the base and *d*D is the altitude. A*d* being perpendicular to AB, and hence vanishing **30°** to the right; and D being thus obtained, the method adopted for finding edge DC is obvious.

2nd Position.—One edge AV of the triangular under face is parallel to and $\frac{3}{4}$" from the P.P., the pyramid vertex V. being directly in front.

After AV has been obtained, we next seek for the **V.P.** of the base edge AB by setting off at **E.** with the Line of Direction the angle $a°$ = BAV. We set it off with the Line of Direction because AB is parallel to the **P.P.**, and hence the Line of Direction represents its Vanishing Parallel. **V.P.** for AB being thus found, and AB measured by its appropriate **M.P.**, we next find corner D of upper base edge by the indirect method adopted in the first position. That is, a line in the **G.P.** perpendicular to AB is drawn and measured = A*d*, and through this the altitude line *d*D is erected.

As will be seen, the **V.P.** for A*d* is obtained at **E.** by a **90°** Vanishing Parallel to Vanishing Parallel AB; and, as this **V.P.** is beyond the paper, its **M.P.** had better be obtained by bisecting the angle at **E.**

PLATE 7.

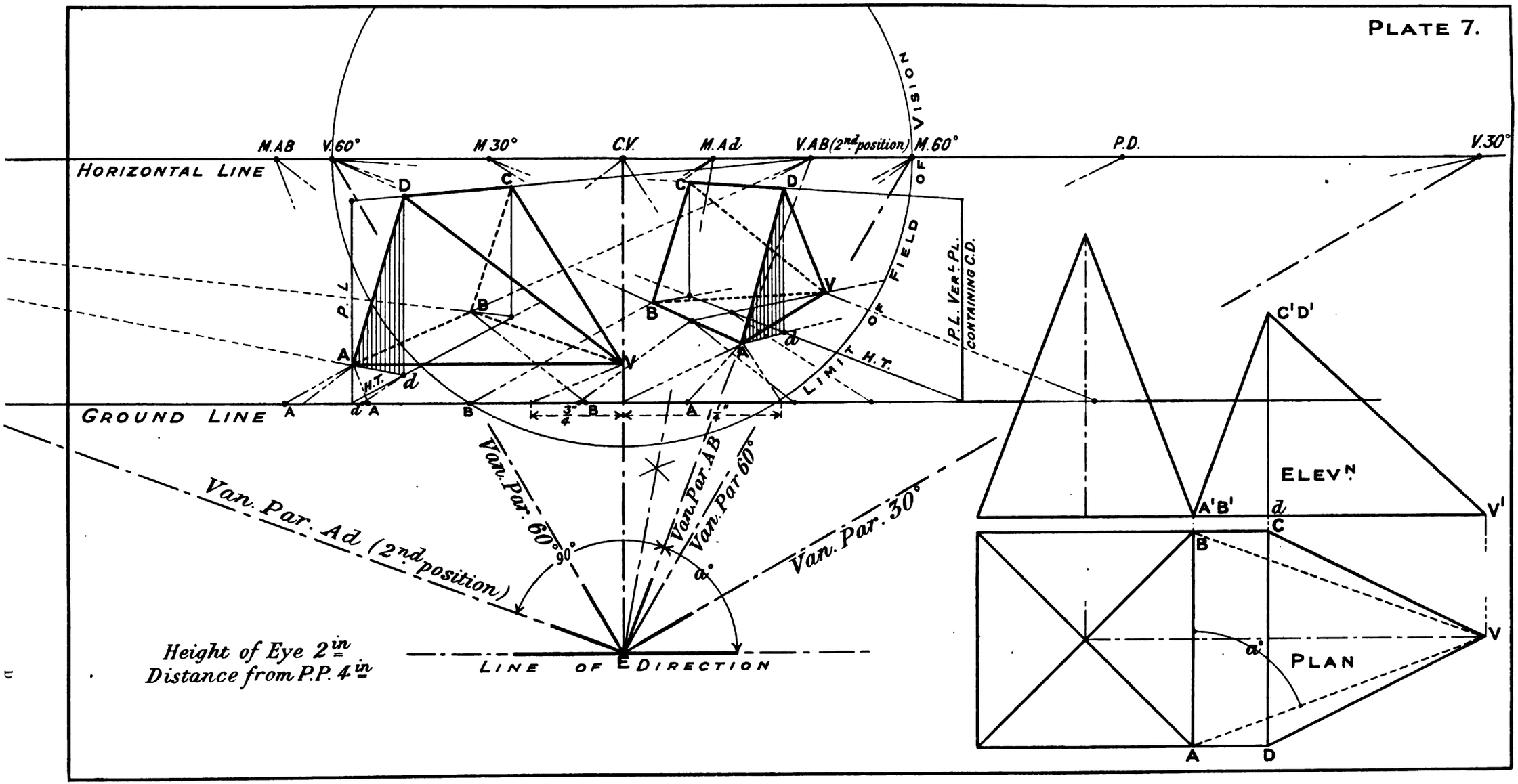

M.AB
V.60°
M.30°
C.V.
M.Ad
V.AB(2nd position)
M.60°
P.D.
V.30°
HORIZONTAL LINE
GROUND LINE
FIELD OF VISION
P.L.
H.T.
LIMIT OF H.T.
P.L. VERT. PL. CONTAINING C.D.
A
B
C
D
V
d
3/4"
1 1/4"
Van. Par. Ad (2nd position)
Van. Par. 60°
Van. Par. AB
Van. Par. 60°
Van. Par. 30°
90°
a°
LINE OF DIRECTION
Height of Eye 2 in
Distance from P.P. 4 in
ELEVN
C'D'
A'B'
V'
PLAN

PLATE VIII.

A CYLINDER WITH AXIS HORIZONTAL AND A CONE LYING ON A HORIZONTAL PLANE.

1st Problem.—A cylinder 3″ diameter and $4\frac{1}{2}$″ long lies on a horizontal plane, which is 4″ below the level of the eye, its line of contact with this horizontal or G.P., vanishing 45° to the left; and so the cylinder may be described as having its axis horizontal $2\frac{1}{2}$″ below the eye and inclined 45° to the left. Near end of line of contact, and therefore near end of axis, 2″ to left and 3″ within the picture.

As in obtaining the perspective of a circle, we enclose it in a square, the middle points of whose edges, together with the cutting points of the diagonals, are sufficient indications for drawing the freehand ellipse (fewer or more points for guidance may be determined on according to the ability of the student).

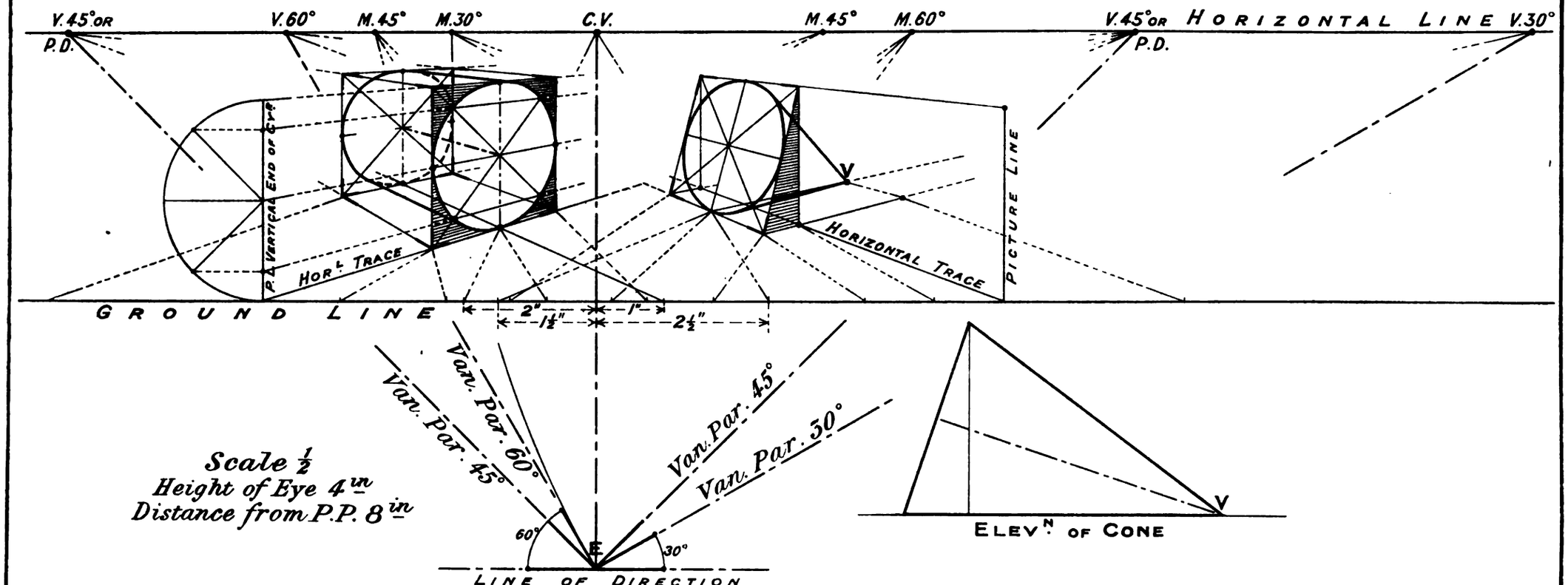
So with a cylinder, we enclose it in a square prism; and as the line of contact of cylinder with **G.P.** is given, the ends of this line will be the middle points of the under edges of the ends of prism.

2nd Problem.—The elevation of a right cone is shown when it rests on its slant side; its base is 3″ diameter, its altitude (length of its axis) $4\frac{1}{2}$″, line of contact with the G.P. vanishing 30° to the right, and near end of it $2\frac{1}{2}$″ to the right and 4″ within the picture.

As a square prism is the most fitting enclose for a cylinder, so here the cone is enclosed in a square pyramid; or rather—without assuming the pyramid—we can enclose the circular base in a square, and obtain the square in perspective by its *overhang*, as in Plate VII.

Tangents to the resulting ellipse from the vertex on the **G.P.** will complete the cone.

PLATE 8.



Scale ½
Height of Eye 4 in.
Distance from P.P. 8 in.

PLATE IX.

PLANES PERPENDICULAR TO THE P.P.

In dealing with lines on the **G.P.**, or with horizontal lines in general, we find that when inclined to the **P.P.** their **V.P.'s** all lie in the **H.L.**; and hence the **H.L.** is the vanishing line for all horizontal planes.

As in the rule laid down for obtaining the **V.P.'s** for lines, so with planes—which are an aggregate of straight lines—and their **V.L.'s.**

Rule for finding **V.L.'s** of planes—

A plane parallel to an original plane passing through E. and produced to cut the P.P. will give the V.L. of any original plane, or set of parallel planes. This, in the case of vertical planes perpendicular to the **P.P.**, will give a line, perpendicular to the **H.L.**, passing through **C.V.** for their **V.L.**

Problem.—Required the perspective representation of a square lying in a plane perpendicular to the P.P. 2″ to the left of the centre; the under corner A of the square—which is $1\frac{1}{2}$″ edge—rests on a horizontal or G.P. 2″ below E. A is also $2\frac{1}{2}$″ from the P.P., and the sides are inclined 60° and 30° to the horizon, as shown in the side elevation. This side elevation has been added in order to explain more graphically the working out of the problem.

First, it should be clearly kept in view that **E.**, the position of the spectator's eye, is directly in front of **C.V.**, and therefore, if it were properly *projected* on to the plane of the picture, it would evidently coincide with **C.V.** It has, however, in the preceding problems, been turned over on the **V.L.** of the horizontal plane through it—that is, the **H.L.**—into the **P.P.**, in order to properly determine the vanishing parallels which go from **E.** to the **P.P.**, because all the **V.P.'s** already sought for have been for horizontal lines.

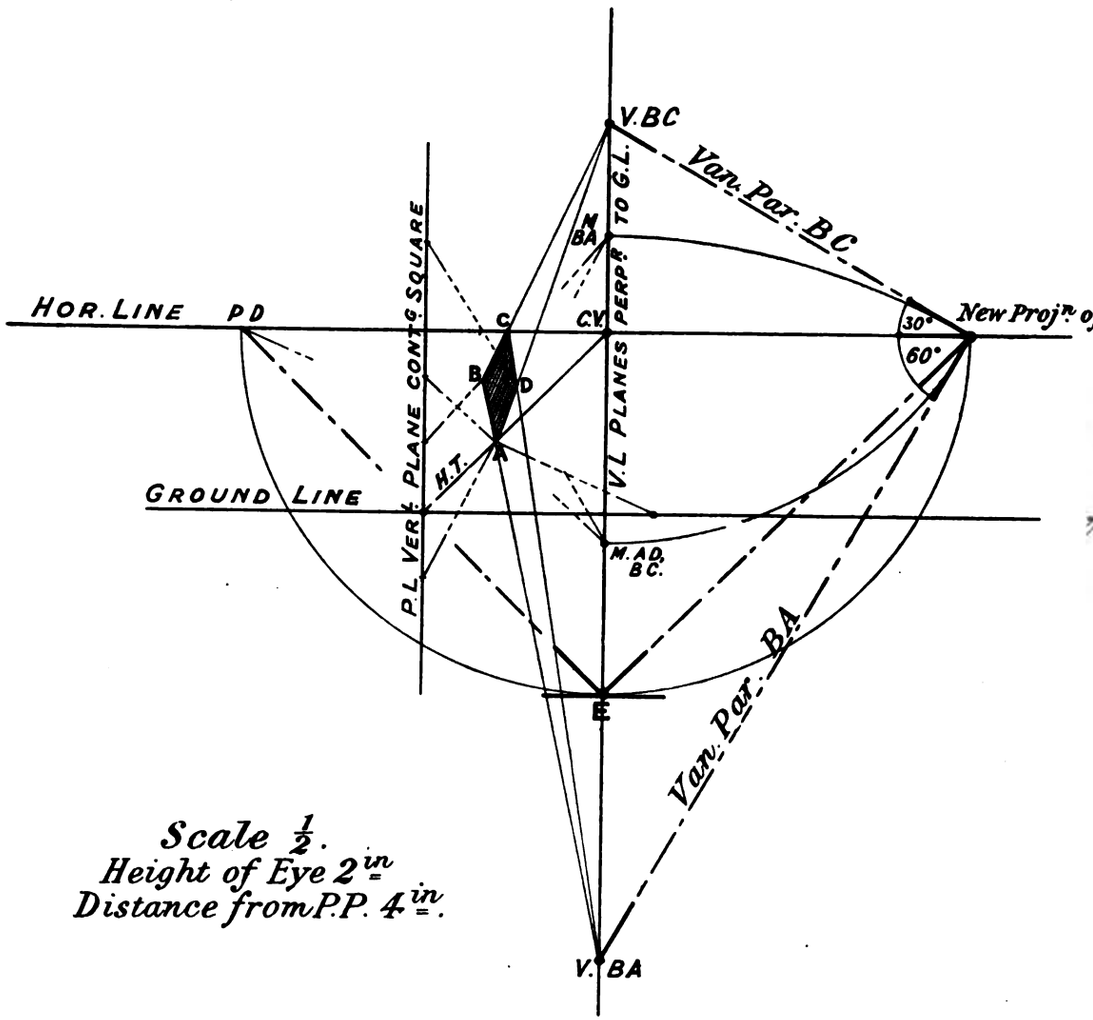
Here, since we require the **V.P.'s** for the edges of the square, which we know will be found in the **V.L.** of its plane, we turn **E.** into the plane of the picture by revolving the vanishing parallel plane (here assumed as giving the **V.L.** for the plane of the square) over on its **V.L.** into the **P.P.** **E.** will therefore fall on the **P.D.** right or left according as the turning movement is to the right or left, and from this *new projection* of **E.** we are enabled to draw the required vanishing parallels **30°** upwards and **60°** downwards, on each side of the horizontal, and so obtain the required **V.P.'s.**

Now, as regards the working out of the problem, corner A is first obtained 2″ to the left and $2\frac{1}{2}$″ within the picture. The **H.T.** of the vertical plane containing the square—that is, the line from **C.V.** through A, produced to meet the **G.L.**—will indicate where the **P.L.** of the plane will be drawn.

Note here that **the P.L. and V.L. of a plane must be parallel lines, because they are intersections with the P.P. of parallel planes, viz. an original plane and its vanishing parallel.**

To measure these edges, having now obtained their **V.P.'s**: If a Line of Direction were drawn through the projection of **E.** on the **H.L.** parallel to the **V.L.** of the plane of the square, we could bisect the angle, as already explained with reference to measuring points for horizontal lines. We find them, however, more directly by setting off on the **V.L.**, and from the **V.P.**, the length of the vanishing parallel for the required line—that is, **V.P.** to **M.P.** for any required line is equal to **V.P.** to **E.**; and having obtained the required **M.P.'s**, measure on the **P.L.** of the square, as has already been done on the **G.L.** (or its **P.L.**) when a horizontal line is to be measured.

PLATE 9.



Scale $\frac{1}{2}$.
Height of Eye 2 in.
Distance from P.P. 4 in.

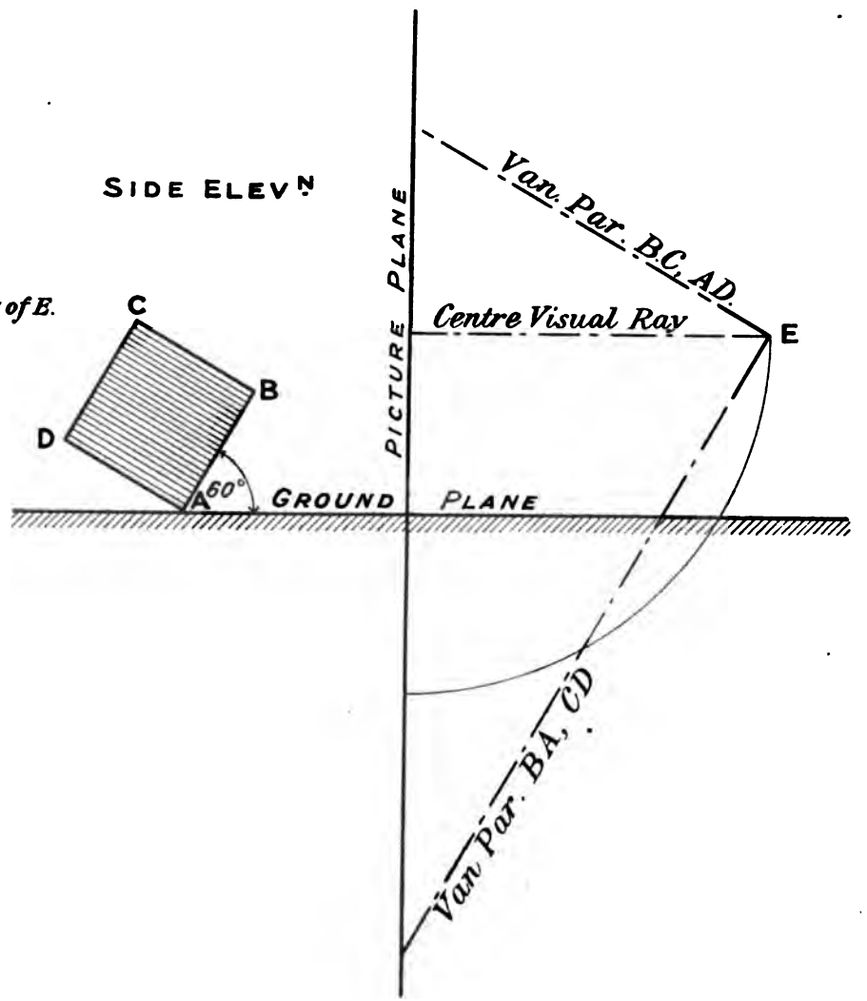


PLATE X.

A RECTANGULAR BOX WITH OPEN LID.

A box $5\frac{1}{4}'' \times 4''$—thickness of material $\frac{5}{8}''$, height $2\frac{1}{4}''$—including lid, when shut, $3\frac{3}{8}''$—lid open at 45°—lies in a horizontal or ground plane $4\frac{3}{4}''$ below the level of the eye. Nearest corner $\frac{1}{2}''$ to the right and $3\frac{3}{8}''$ within the picture. Long edges of box vanish 30° to the right.

Obtaining the perspective of the box will present no new difficulty, being similar to the example in Plate V.

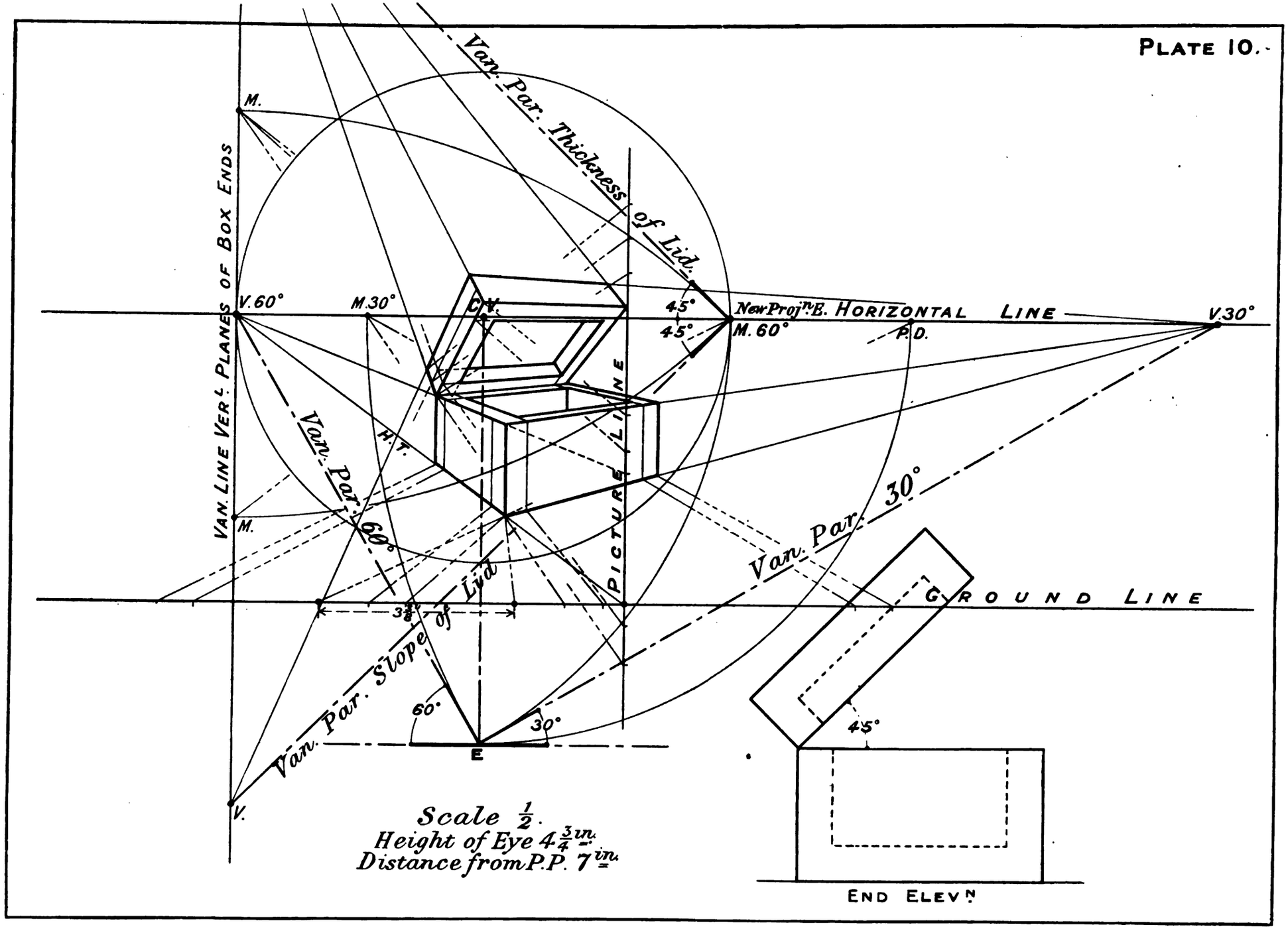
It is only when we come to the treatment of the lid, and the application of the rule learnt in the last example, that any novelty occurs.

Producing the bottom edge—the **H.T.** of the vertical side of the box, which vanishes 60° to the left—we obtain, on the **G.L.**, the point through which its **P.L.** is drawn, and on the **H.L.** at **V.P.** 60° its **V.L.**

And since the lid in its movement from a shut position upwards will revolve in this vertical plane, the **V.P.'s** for its edges—its thickness and width edges—must be sought for in the **V.L.** of its plane. To find these we must obtain the new projection of **E.**, which, when the vanishing parallel plane of the box-end is revolved on its **V.L.**, will fall on the **H.L.** at **V.P.** 60° on the right (**M.P.** for 60° left). Setting off here 45° above and 45° below the horizontal, we find the **V.P.'s** required; and with these **V.P.'s** as centres and distances to **E.** obtain the associated **M.P.'s.**

Thickness and width edges of lid being drawn to their **V.P.'s** are measured on the **P.L.**, as has already been explained. Long edges of lid being still parallel to length of box will vanish 30° right. We have thus found a *direct* method of determining lines not horizontal, which is certainly preferable to the *indirect* and merely mechanical treatment explained in Plate VII.

PLATE 10.



Scale $\frac{1}{2}$.
Height of Eye $4\frac{3}{4}$ in.
Distance from P.P. 7 in.

PLATE XI.

PLANES IN VARIOUS POSITIONS

An original plane may be in any of the following positions as regards its relation to the horizontal and the **P.P.**

1. It may be a Vertical plane and perpendicular to the **P.P.**, as illustrated in Plate IX.; or it may be a vertical plane, but not perpendicular to the **P.P.**, as in Plate X.

In both cases the angle between its **H.T.** and the **G.L.** will equal the angle between the original plane and the **P.P.**; and its **V.L.** and **P.L.** will both be perpendicular to the **H.L.**

2. It may be an Inclined plane; that is, it may be inclined to the **G.P.**, though perpendicular to the **P.P.**

In this case its **H.T.** will be perpendicular to the **G.L.**, and both **V.L.** and **P.L.** will make angles with the **H.L.** equal to the inclination of the original plane to the **G.P.**

3. It may have its **H.T.** parallel to the **P.P.** (*i.e.* the **G.L.**), and, not being vertical, it may in rising upwards recede from the **P.P.**, or it may advance towards the **P.P.**

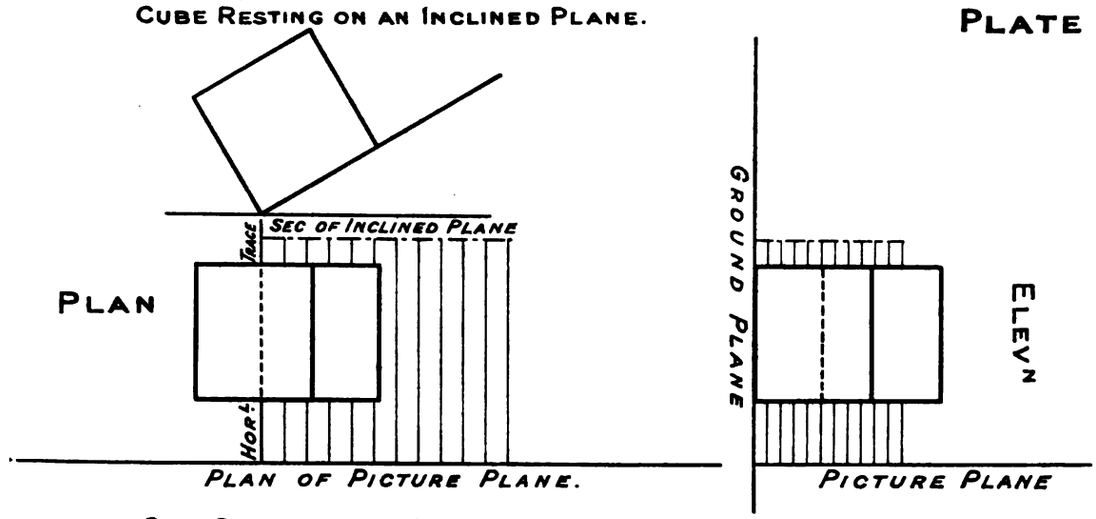
In the first case we call it an Ascending plane, in the second case a Descending plane (in the latter case because, as it recedes from the **P.P.**, it descends).

In both cases, as will be seen in the side elevations, the sum of its inclinations to the **G.P.** and the **P.P.** will equal **90°**.

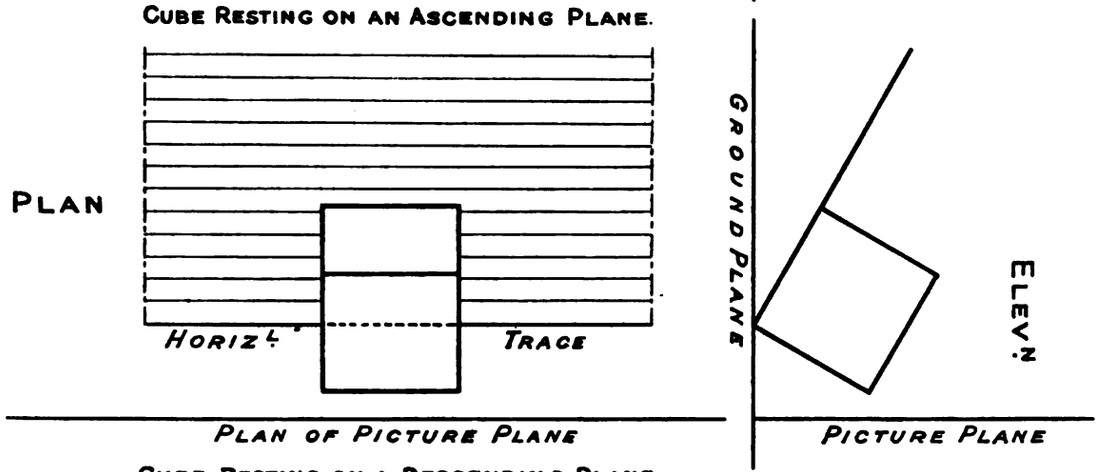
4. It may be an Oblique plane; that is, while inclined to the **G.P.**, its **H.T.** may be also inclined to the **P.P.**, and therefore the angle between its **V.L.** (or **P.L.**) and the **H.L.** will not be equal to the inclination of the original plane to the horizontal.

PLATE II.

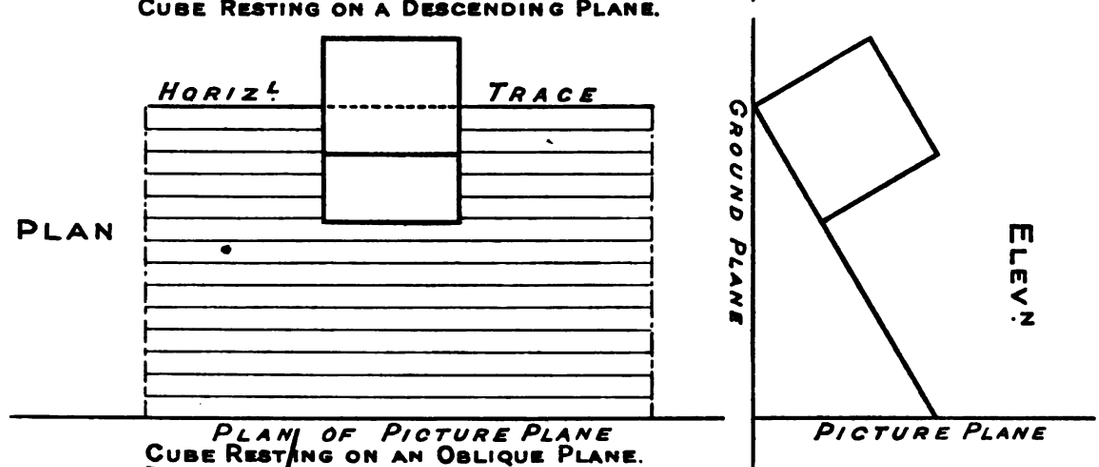
CUBE RESTING ON AN INCLINED PLANE.



CUBE RESTING ON AN ASCENDING PLANE.



CUBE RESTING ON A DESCENDING PLANE.



CUBE RESTING ON AN OBLIQUE PLANE.

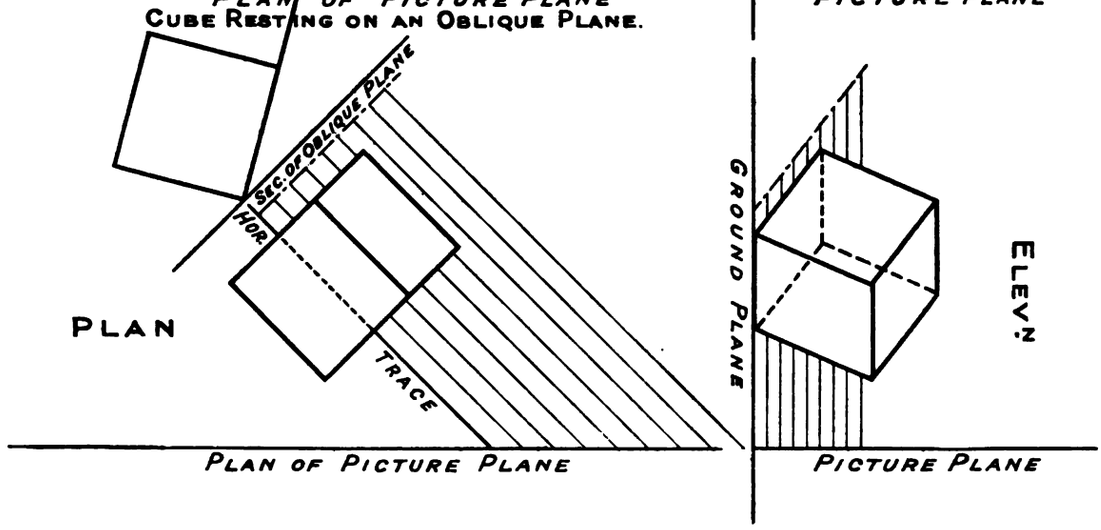


PLATE XII.

PYRAMID ON A PLANE INCLINED TO THE G.P. AND PERPENDICULAR TO THE P.P.

A pyramid—its base square of 3″ edge, altitude 6″—rests on an inclined plane, whose H.T. is 2″ to the right, and its inclination to the horizon or the G.P. 30°. The lowest corner of the pyramid base is 3″ along this H.T., and the base edges are inclined 50° and 40° to it.

Because the plane of the base is perpendicular to the **P.P.** and inclined **30°** to the **G.P.**, its **V.L.** will pass through **C.V.** and will make **30°** with the **H.L.**

And because it meets the **G.P.** 2″ to the right (its **H.T.** is 2″ to the right), we measure along the **G.L.** 2″ to the right and draw through it the **P.L.** parallel to the **V.L.**

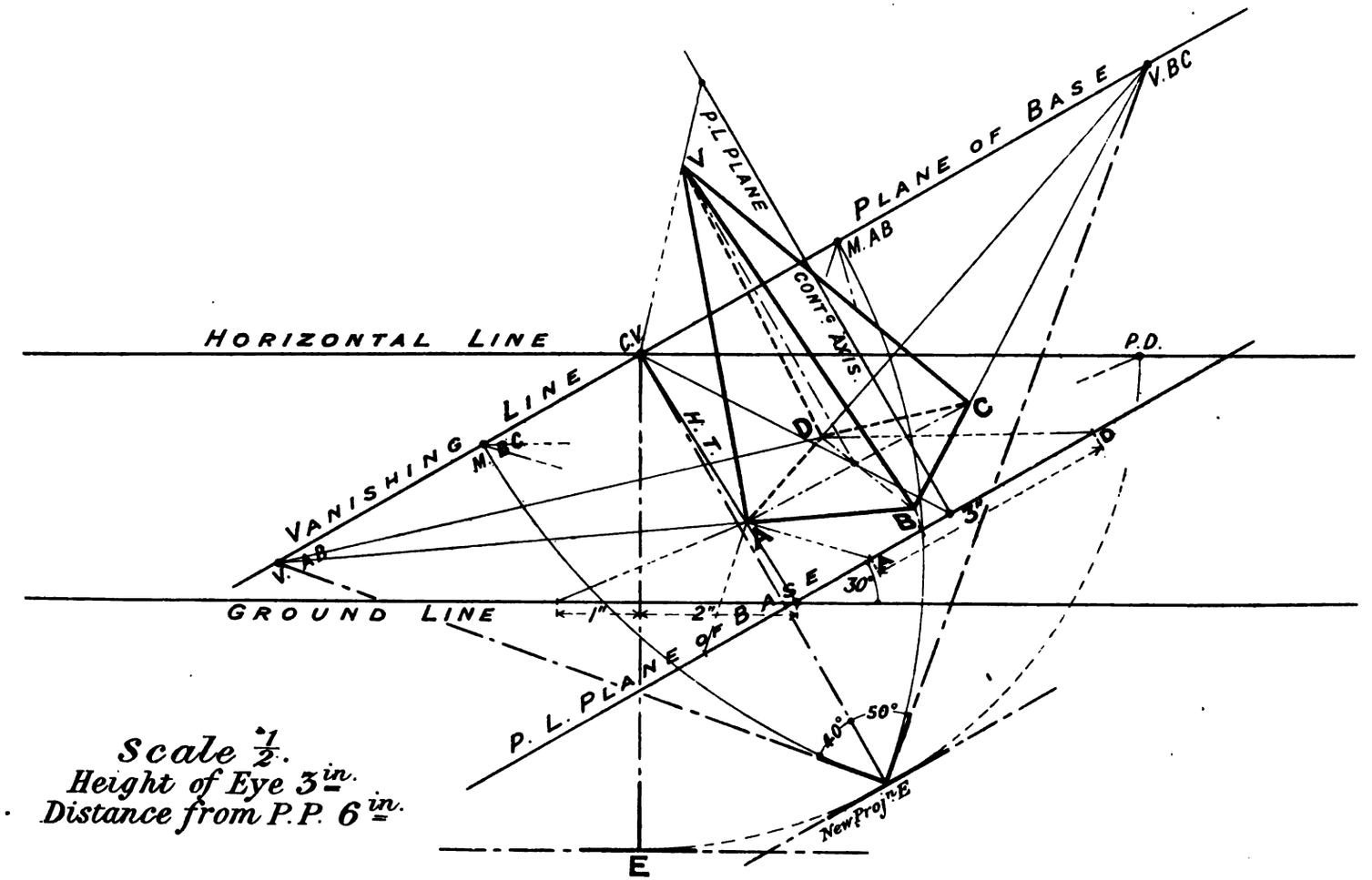
For lowest corner A, measure along the **H.T.** 3″; and in order to obtain **V.P.'s** for base edges, fold the vanishing parallel plane over on its **V.L.** as a hinge, carrying **E.** with it, into the **P.P.** This means setting off at **C.V.** a line perpendicular to the **V.L.** of the inclined plane, and making it equal to the distance **E.** is from the **P.P.**

This line **C.V.** to new projection **E.** is actually a projection on the **P.P.** of the vanishing parallel for the **H.T.** of the inclined plane, and because the edges of the square base make angles of **50°** and **40°** with the **H.T.**, set off these angles at new projection **E.** on each side of this vanishing parallel; noticing that **40°** is the inclination of the under near edge.

The obtaining of the **M.P.'s**, and completion of the base, will be understood from what has already been done in previous examples.

As regards the vertex of the pyramid, its axis will be parallel to the **P.P.**, and therefore may be measured by any plane perpendicular to the base plane which passes through the axis. In the example a line has been brought from the **C.V.** through the centre of the base (bottom end of axis) forward to meet the **P.L.**, and through it a line has been drawn perpendicular to the **P.L.** The axis length set off on it and returned to the **C.V.** will cut off the perspective length and determine the vertex.

PLATE 12.



Scale $\frac{1}{2}$.
Height of Eye $3\frac{in.}{=}$
Distance from P.P. $6\frac{in.}{=}$

PLATE XIII.

CUBE RESTING ON AN ASCENDING PLANE.

The under face of a cube of 2″ edge rests on a plane which ascends from the G.P. at 40°. Its H.T. is parallel to and 2½″ from the G.L. Edges of base of cube equally inclined, 45°, to the H.T. of its plane, and lowest corner A in the H.T. 2″ to the right.

A side elevation has been drawn indicating the position of the ascending plane, sloping up from its **H.T.** at **40°**; and the eye being seen here in its proper position, as regards the **P.P.**, we have merely to draw through it a parallel to the ascending plane to determine the **V.L.** of that plane.

By drawing also through **E.** a line perpendicular to this, we obtain the **V.P.** for lines perpendicular to the ascending plane; that is, for those edges of the cube rising from its under face.

In the perspective diagram, therefore, and keeping this side elevation process in view, we obtain the **V.L.** for the ascending plane, by setting off at either **P.D.** a line making **40°** with the **H.L.** to meet the perpendicular through **C.V.**

This, then, is the **V.P.** for all lines lying on the ascending plane which are perpendicular to its **H.T.**, that is, inclined at same angle to the **G.P.**

Next, as the **H.T.** is parallel to the **G.L.**, we now draw its **V.L.** parallel to the **G.** or **H.L.**

To obtain the **P.L.**, the side elevation shows the ascending plane produced below ground to meet the **P.P.**; so, in a similar way, and as we are simply compounding this side elevation with the usual perspective diagram, we set off to left of centre of **G.L.** 2½″—the distance of **H.T.** from **G.L.**—and through it draw a line parallel to the vanishing parallel till it meets the same perpendicular through **C.V.** Through this another horizontal line will represent the **P.L.** required.

Corner A being found in the **H.T.** 2″ to right of centre, we have next to find the **V.P.'s** for the base edges which are equally inclined to the **H.T.** To determine these, fold the vanishing parallel plane on its **V.L.** as a hinge into the **P.P.**, and from the *new projection* **E.** set off the required vanishing parallels, and so obtain the **V.P.'s**; and also, as before, find the **M.P.'s** and measure off sides of square on the **P.L.**

Having thus completed that face of the cube which rests on the ascending plane, we deal next with the edges rising from this. These, being perpendicular to the base, vanish to a point directly below the **C.V.**, which **V.P.** is obtained, as shown in the side elevation (the same method being reproduced in the perspective diagram, however).

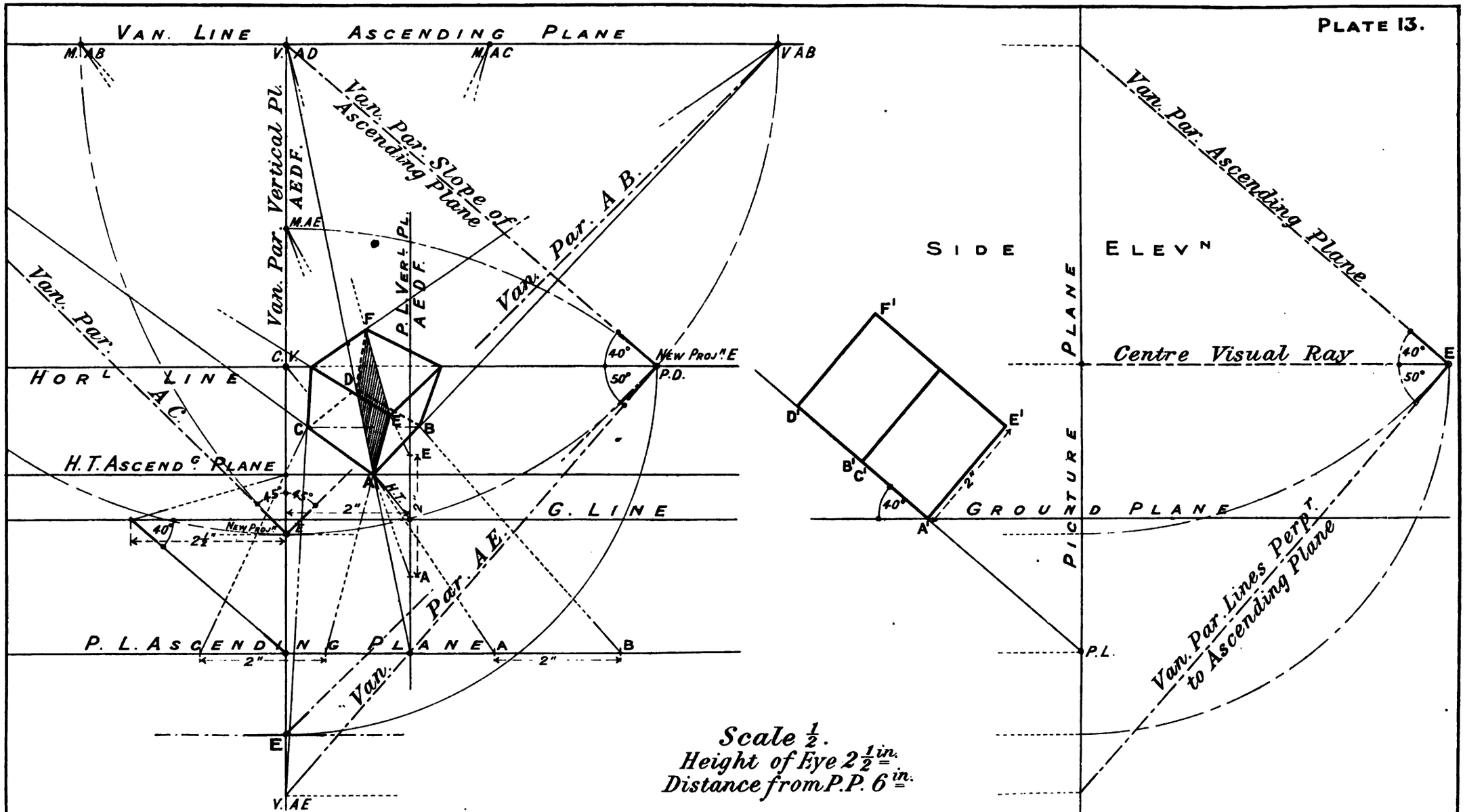
To measure these lines, that is, to measure some one of them, such as edge AE, consider AE as being in a vertical plane perpendicular to the base or ascending plane, and therefore perpendicular to the **P.P.**

This vertical plane in this particular case, where the base edges are equally inclined to the **H.T.**, will bisect the cube (its bisection is indicated by shade-lines), and, being perpendicular to the **P.P.**, its **V.L.** will be the perpendicular through **C.V.**

To find its **P.L.** we can draw its **H.T.**—that is, **C.V.** is joined to A and produced to meet the **G.L.**, or its intersection with the ascending plane (the diagonal AD), produced to meet the **P.L.**, will give a point common to the two **P.L.'s.** And so through it, or through the intersection of its **H.T.** with the **G.L.**, draw the perpendicular line, which will be the **P.L.** required.

M.P. for edge AE is found in the usual way, and AE measured on the **P.L.**

The completion of the cube need not be further explained.



Scale $\frac{1}{2}$.
Height of Eye $2\frac{1}{2}$ in.
Distance from P.P. 6 in.

PLATE XIV.

Shows an Isometric treatment of this Problem.

PLATE 14.

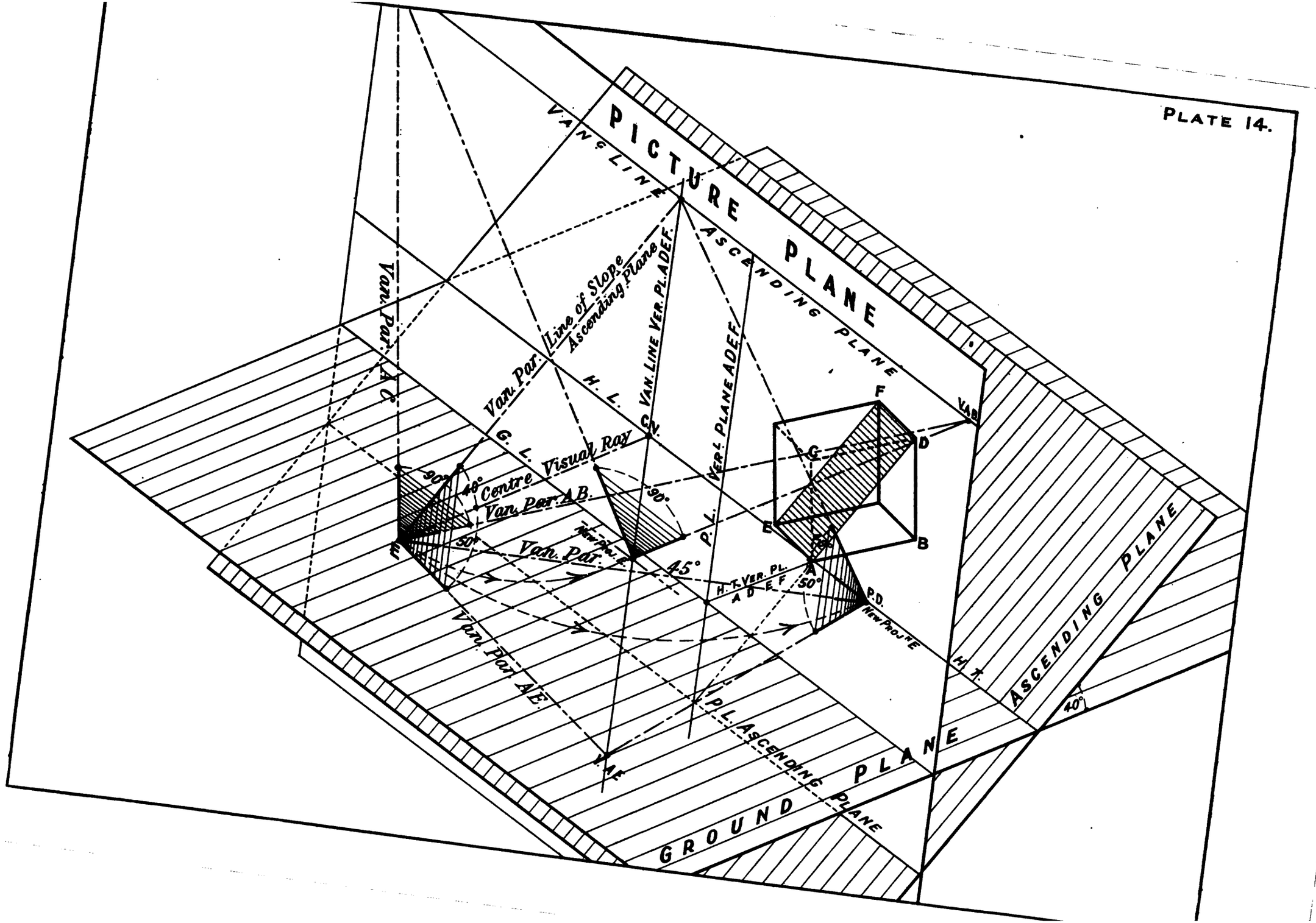

PICTURE PLANE
VAN. LINE
ASCENDING PLANE
Van. Par. AE
Van. Par. Line of Slope Ascending Plane
VAN. LINE VER. PL. ADEF.
NOR. PLANE ADEF
H. L.
G. L.
Centre Visual Ray
Van. Par. AB.
Van. Par.
P.L.
H.T. VER. PL. A D E F
P.D.
New Proj.ne
H. T.
ASCENDING PLANE
P.L. ASCENDING PLANE
GROUND PLANE
90°
40°
50°
45°
70°
F
D
C
E
B
A
VAB
AE

PLATE XV.

A WRITING-DESK ON AN ASCENDING PLANE.

The side elevation of a writing-desk with lid open is shown. Scale ¼.

The perspective of it is required when it rests on an ascending plane inclined 40°, the H.T. of which is 6″ from the G.L.

The lowest corner A of the box is on the H.T. and 4″ to the right, and the edges of its base are equally inclined to the H.T.

We find, as in the last problem, the **V.L.** and **P.L.** of the ascending plane, together with the **V.P.** for perpendiculars to it; and then again, by revolving its vanishing parallel plane into the **P.P.**, we find from the new projection of **E.** the **V.P.'s** for base edges AB and AC, and measure its width and length, 9″ and 14″ respectively. Joining these to the **V.P.** AD, we next have to measure the varying lengths of the uprights, as AD, BE.

Assuming, as in the last, a vertical plane containing AD, we find its **P.L.**; and having also obtained the **M.P.** for AD, we measure not only AD but also BE on the same front edge. The thickness BE can then be transferred to the back edge by a parallel to AB—a line to **V.P.** AB.

Front top edge through D will vanish to **V.P.** AC, and so with the hinge edge EK. Complete by finding edge parallel to DE, which could have been done by completing the rectangular base and bringing up a corresponding line to BE from the invisible corner. A better—more artistic—way will be by finding **V.P.** for DE; and this will be found on the **V.L.** for the plane of the end—that is, plane ADBE. Now, a **V.L.** for a plane must contain the **V.P.'s** for all lines in it, and so, knowing the **V.P.'s** for two lines in this plane, we can determine the **V.L.** itself.

This is found by joining **V.P.** AB to **V.P.** AD. And so **V.P.** for edge DE is sought for on this **V.L.**, and utilized for completing rectangular visible surface of box. As regards the lid, its near end lies in the same plane as the box end ADBE. Back edge GF is perpendicular to the plane of the base, and so will vanish to **V.P.** AD. GE is simply a continuation of the line parallel to the base through **E.** The projection of GF beyond back edge of box, together with the parallel to FG through H on to the base edge AB produced, are measured (**M.P.** AB and **P.L.** ascending plane), and then from **V.P.** AD upwards.

As regards length of GF, an interesting point comes in here. If it were possible, that is, if edge AD were sufficiently far off from the **P.P.**, we might measure the required length from D upwards on it, and so transfer it by a line to **V.P.** AB across to cut GF. This, as will be seen, is not in our power, because by producing AD we find it cuts the **P.L.** (and therefore the **P.P.**) of the vertical plane containing it lower down than F would evidently reach to. We have, consequently, to obtain a *new vertical* plane containing GF, and utilize its **P.L.** for the purpose of measurement.

FH again will vanish to **V.P.** AB, and HE joined will complete the near end of the lid. As regards the far end and the edge through K, corresponding to EH, find the **V.P.** for EH in **V.L.** of the plane of the end, and join to K.

PLATE 15.

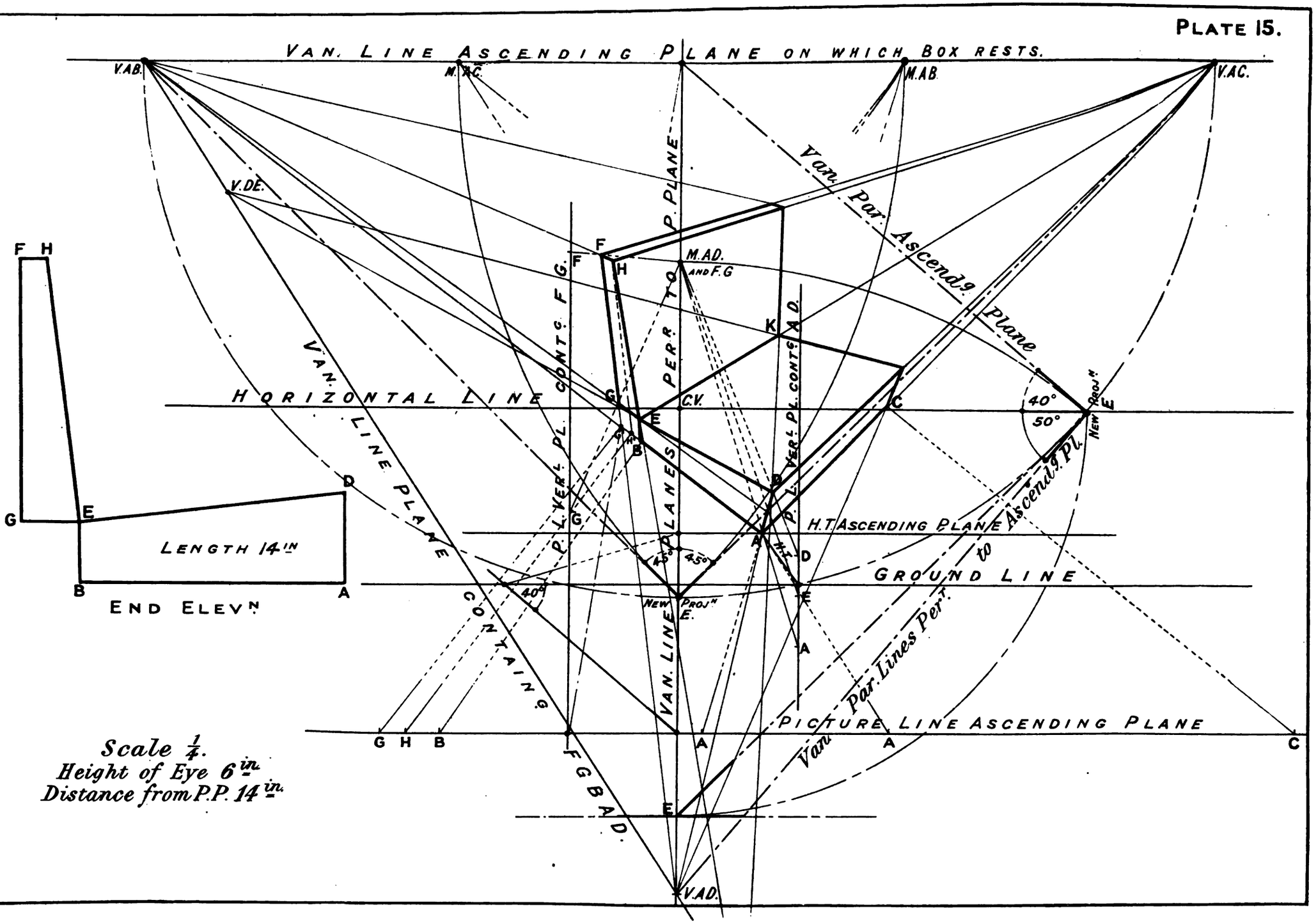

VAN. LINE ASCENDING PLANE ON WHICH BOX RESTS.
V.AB.
M.AC.
M.AB.
V.AC.
V.DE.
P. PLANE
Van. Par. Ascend.g Plane
M.AD. AND F.G
PERP. TO
VER.L PL. CONTG A.D.
VER.L PL. CONTG F.G.
HORIZONTAL LINE
C.V.
40°
50°
NEW PROJN E
VAN. LINE PLANE
PLANES
H.T. ASCENDING PLANE
Ascend.g Pl. to GROUND LINE
45°
45°
40°
NEW PROJN E.
VAN. LINE
CONTAING
Van. Par. Lines Perp.
PICTURE LINE ASCENDING PLANE
F G B A D.
V.AD.
F H
G
E
D
B
A
LENGTH 14 IN
END ELEVN
Scale 1/4.
Height of Eye 6 in.
Distance from P.P. 14 in.

PLATE XVI.

HEXAGONAL PYRAMID WITH BASE ON AN OBLIQUE PLANE.

A hexagonal pyramid, base edges 2″, altitude 4″, lies in an oblique plane, inclined at 45° to the horizontal or ground plane.

The under edge of the pyramid base rests on the G.P. (in the H.T. of the oblique plane), near end of it 5″ along the H.T.

The H.T. starts from a point on the G.L. 4″ to the left, and vanishes at 30° to the right.

Having found the **H.T.** according to the specification of its position, we next find the **V.L.** of the oblique plane, and this is best obtained by remembering that all lines in an inclined or oblique plane which are perpendicular to the **H.T.** of the plane will be inclined to the horizontal at the same angle as the oblique plane. That is, assuming any vertical plane cutting the **H.T.** of the oblique plane at right angles, then its intersection with the oblique plane will be a measure of the inclination of that plane—it will be a line inclined at the same angle. So that to find the **V.L.** for any oblique plane we find the **V.P.** for the line of its slope, and join it to the **V.P.** for its horizontal trend.

Now, without absolutely indicating any specific vertical plane to fulfil the above conditions, we know that its **V.L.** will be a perpendicular through 60° on the left.

The vanishing parallel plane revolved over on this **V.L.** will project **E.** into the **H.L.** at **M.P. 60°** on the right, and setting off here **45°** upwards with the **HL.**, we obtain the **V.P.** for the slope, which joined to **V.P. 30°** (**V.P.** for **H.T.**) will give **V.L.** for oblique plane.

AB, under edge of base, is next measured along **H.T.**, and to complete the hexagon without the use of further **V.P.'s.** Enclose it in a rectangle—that is, measure beyond AB on the **H.T.** distances equal to the projection of F and C beyond the base, and through these points, as well as A and B, draw lines to the **V.P.** for the slope. To measure distances on these lines upwards, we select any suitable vertical plane containing any one of the lines to be measured. The most appropriate seems to be that bisecting the base, and therefore containing the axis; that is, bisect AB, trace down from **V.P.** AE, etc., a line on the oblique plane to meet this bisection, and then from **V.P. 60°** forward on the **G.P.** will be the **H.T.** of this same vertical plane, which, cutting the **G.L.**, will enable us to obtain the **P.L.**

Find next **M.P.** on the **V.L.** of the vertical plane, and cut off distances on middle line of hexagon equal to 10 and 12.

The **V.P.** for axis lies in the same **V.L.**, and will be found by a perpendicular to the vanishing parallel for the slope: that is **45°** below the **H.L.**; and it is measured also in the usual way.

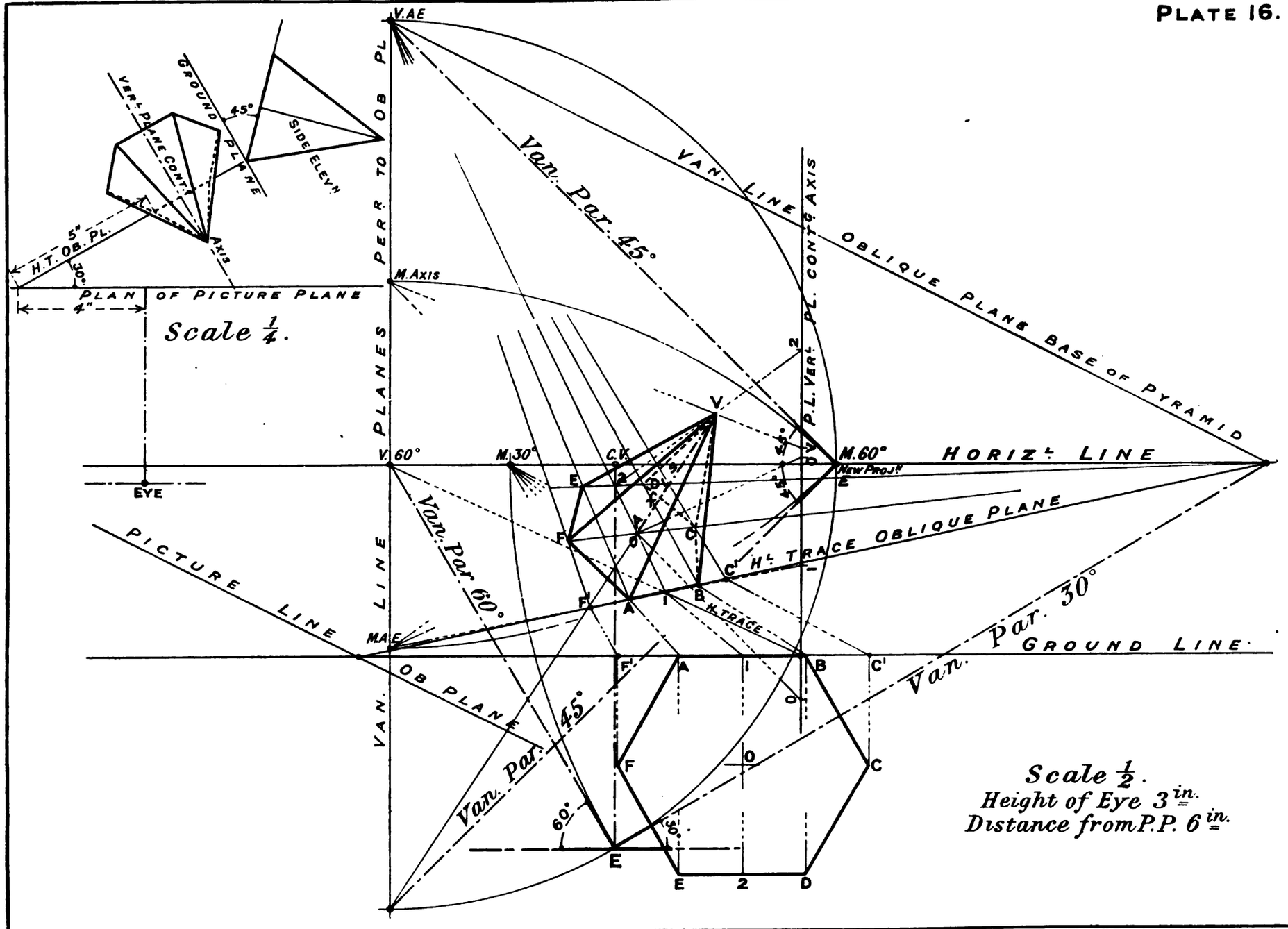

V.AE
VAN. LINE
OBLIQUE PLANE BASE OF PYRAMID
Van. Par. 45°
PERP. TO OB. PL.
PLANES
P.L. VERT. PL. CONTG AXIS
M.AXIS
V. 60°
M. 30°
C.K.
M. 60°
HORIZ. LINE
New Proj.n
EYE
Van. Par. 60°
PICTURE LINE
H. TRACE OBLIQUE PLANE
H. TRACE
M.A.E.
GROUND LINE
VAN. OB PLANE
Van. Par. 45°
Van. Par. 30°
VER. PLANE CONTN
GROUND PLANE
SIDE ELEVN
45°
AXIS
5" H.T. OB. PL.
30°
PLAN OF PICTURE PLANE
4"
Scale 1/4.
Scale 1/2.
Height of Eye 3 in.
Distance from P.P. 6 in.

PLATE XVII.

A CUBE ON AN OBLIQUE PLANE WITH ITS EDGES INCLINED TO ITS H.T.

A cube of 3″ edge lies on an oblique plane inclined at 40°. Its H.T. vanishes 40° to the left, and proceeds from a point in the G.L. 4″ on the right. Lowest corner of cube 4″ along this H.T., and lowest edge of cube makes an angle of 15° with the H.T.

The mode of determining the **H.T.** and the **V.L.** for the oblique plane is the same as in the previous problem.

Assume any plane perpendicular to the **H.T.**, and from the new projection of the eye for it—the **M.P. 50°** on the left—set off above **H.L.** the vanishing parallel for the slope of oblique plane; that is, for a line in this vertical plane, making **40°** with the horizontal. This **V.P.**, together with **V.P.** for **H.T.**, will give **V.L.** for the oblique plane.

To obtain **V.P.'s** in this **V.L.** for the base edges, we must obtain another projection of **E.** on to the plane of the picture. That is, the vanishing parallel plane for the oblique plane is again turned over on its **V.L.**, and so a new projection of **E.** will be found along the perpendicular line passing through **C.V.L.**

Before attempting to find where, on this line, we must place the new projection of **E.**, we had better turn to the small diagram, which illustrates what is being done.

As has already been stated, the proper projection of **E.**—with reference to its relation to the **P.P.**—coincides with **C.V.**; and so with regard to the various vanishing parallel lines from **E.** to determine the **V.P.'s.**

The vanishing parallel plane for the oblique plane, therefore, or as much as lies within the vanishing parallel for the **H.T.**, and the vanishing parallel for the slope, will be represented by the shaded triangle. So that in order to fold the plane of this triangle over into the plane of the paper, we may draw through **C.V.** a perpendicular to the **V.L.**, indicating thus the centre—the **C.V.L.**, and, finding the *true* length of this line, set it off from **C.V.L.** along the perpendicular. This, of course, will give the new place or projection of **E.** for the plane we are dealing with.

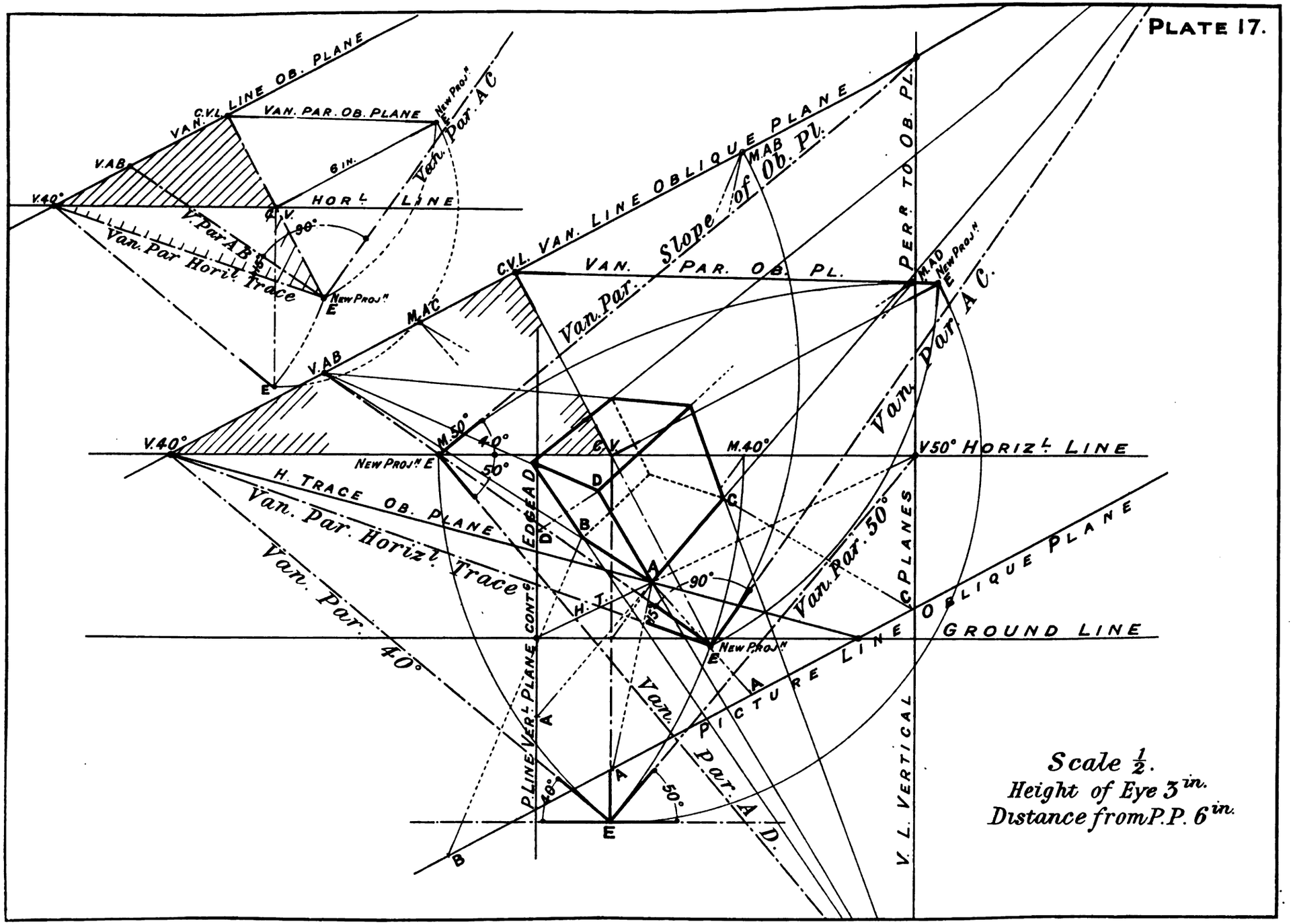
Or we may find it by setting off from **V.P. 40°** the length of the vanishing parallel **40°**, or at **V.P.** of the slope, setting off the vanishing parallel of the slope.

All three lengths should agree in the one point. From this third projection of **E.** by joining to **V.P. 40°**—that is, **V.P.** for **H.T.**—and setting off **15°** to right of it, we obtain **V.P.** AB, under edge of base; and again, **90°** beyond this will give **V.P.** for edge AC. **M.P.'s** are obtained as usual.

The **P.L.** is found by a parallel to the **V.L.**, through the near end of the **H.T.** Upright edges, as AD, will vanish perpendicular to the slope, that is **50°** downwards.

And again, to measure one of them, as AD, determine the vertical plane which will contain it, and its **P.L.** This last is obtained by drawing the **H.T.** across from A—that is, the line from **V.P. 50°** on the right through to meet the **G.L.**—and erecting the perpendicular. The **M.P.** on the **V.L.** of the vertical plane now measures AD.

PLATE 17.



Scale $\frac{1}{2}$.
Height of Eye 3 in.
Distance from P.P. 6 in.

PLATE XVIII.

PROJECTION OF SHADOWS—PARALLEL RAYS—*SUN* IN THE PLANE OF THE PICTURE.

In working out problems dealing with the projection of shadows of objects when in perspective, we deal with rays either parallel or radial.

That is, the source of light in out-of-door studies being the sun, any geometrical treatment of the problem will be on the supposition that the light rays are parallel.

In indoor effects the origin of light, a candle or lamp, being treated as a point, we shall have then to deal with diverging rays, and this results in a radial dissemination of light and projection of shadows.

The position of the sun, too, must be indicated with reference to its relation to the **P.P.** —or to the spectator; and also its altitude or its relation to the **G.P.** With regard to the **P.P.**, it may be in front—that is, behind the spectator; or behind the **P.P.**—that is, in front of the spectator (the spectator, of course, facing the picture); or it may be in the plane of the picture.

Its altitude will be measured in degrees above the horizontal or ground plane. Then, knowing the whereabouts of the sun, to determine the **V.P.** for its rays, and so to define the shadows, we proceed in the usual way explained for finding the **V.P.** of a line, or set of parallel lines: draw through **E.** a line parallel to the rays to meet the **P.P.** When the sun is behind the spectator, to right or left, the vanishing parallel will meet the **P.P.** at an angular distance *below* the horizon equal to the sun's altitude, and on the side opposite to the sun's position.

When the sun is in front, the vanishing parallel will be the line joining **E.** to the sun (taken as a point); and so the **V.P.** for the rays will really correspond with the projected position of the sun.

When the sun is in the plane of the picture there will be no **V.P.** for the rays: they will be parallel lines in the perspective diagram.

In the first case, also, the shadows will recede from the **P.P.** and decrease in size.

In the second case—the sun being in front of the spectator—the shadows will advance towards the **P.P.** and increase in size.

In whatever position the sun may be, the shadow of a plane figure, in a horizontal position, will be a figure equal in size and shape to the original figure causing the shadow, and consequently with its edges vanishing to the same **V.P.'s.**

In the examples given on Plate XVIII. the rays are parallel to the **P.P.**, and inclined **45°** to the horizontal—that is, the sun is assumed to be in the plane of the picture, on the spectator's left, at an altitude of **45°**; and so the rays come down at **45°** towards the right.

Fig. 1 shows, to half the scale of the perspective diagram, a square plane in four different positions with the resultant shadows.

No. 1 shows a vertical square of $1\frac{1}{2}$″ edge, and perpendicular to the P.P., 2″ to the left, near edge, $2\frac{1}{2}$″ within. Because the rays are inclined **45°**, and in planes parallel to the **P.P.**, the shadow will be a square of the same size.

No. 2 is vertical, but in a plane inclined 45° to the P.P., near corner, $\frac{1}{2}$″ to the left, and 4″ within.

No. 3.—Same square inclined 40° to the horizon, its H.T. inclined 45° towards the left, near end of base edge, $1\frac{1}{4}$″ to the right, and 2″ within.

No. 4.—A square of 1″ edge horizontal and 3″ above the level of the eye, edges equally inclined to the P.P. Nearest corner $1\frac{1}{4}$″ to the left and $\frac{3}{4}$″ within.

Treating all these in plan, we find that the shadows of the various corners are found on lines parallel to the **P.P.** and at distances from them equal to their heights.

In the perspective diagram, 1 and 2 having vertical edges, their shadows are parallel to the **G.L.**, and the rays at **45°** through their upper ends will determine their lengths.

The shadows of the top horizontal edges will be lines parallel to them, and so will vanish to the same **V.P.'s.**

This applies to all the four squares.

In position 3 the sloping edges are in **V.** planes inclined **45°** towards the right; and so to obtain shadow of top edge we might drop a perpendicular from one end, and finding the shadow of this upright line (as in positions 1 and 2), so determine that of the top edge, and complete by joining to bottom edge. A better method, however, and, to use a previous phrase, a more artistic method, is to find where the shadow of the sloping edge will vanish to.

The shadow of a line on any plane will be the intersection with that plane of a plane containing the line and a ray of light (the rays being parallel). The **V.P.** for this line then will be the intersection of the **V.L.'s** for the planes which intersect.

Rule for obtaining the V.P. for a shadow-line, knowing the position of the line throwing the shadow and that of the plane receiving it.

Join the V.P. of the line to the V.P. of the parallel rays, thus obtaining the V.L. of a plane containing both; the intersection of this V.L. with the V.L. of the plane receiving the shadow will be the V.P. for the intersection line of both planes, and therefore the V.P. required.

When the rays are parallel to the P.P., and so have no V.P.—do not converge in the perspective diagram—we obtain the V.L. of the plane containing the line and a ray by drawing through the V.P. of the line a parallel to the rays.

When the plane receiving the shadow is parallel to the P.P.—and hence has no V.L.—the shadow-line projected on it will be drawn parallel to the V.L. of the plane containing the line throwing the shadow and the V.P. of the rays. That is the V.L.P.S.

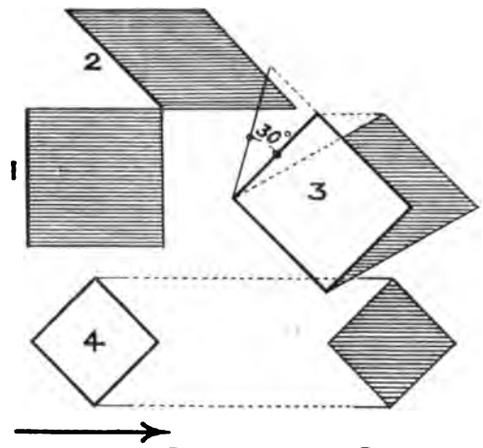
As will be seen in the small isometrical diagram where a vanishing parallel is drawn from **E.** to meet the **P.P.**, and a line drawn also through **E.** parallel to the rays; then, to obtain the vertical trace, or intersection with the **P.P.** of a plane containing both, we simply draw through the *vertical trace* of the vanishing parallel a line parallel to the rays.

In square No. 3, to obtain **V.P.** for shadow of sloping edges, draw through **V.P.** of these edges a line **V.L.P.S.** at **45°** to meet the **H.L.**, which is the **V.L.** of the plane receiving the shadow; this, then, is the **V.P.** for the shadow edges, and rays through the upper ends will determine their lengths. Shadow of 4 is obtained by finding *plan* of horizontal square on the **G.P.**, and drawing through the corners in the *plan* parallels to the **G.L.** to meet rays from the square overhead, thus making use of verticals through the corners and their shadows, just as in squares 1 and 2.

PLATE 18.

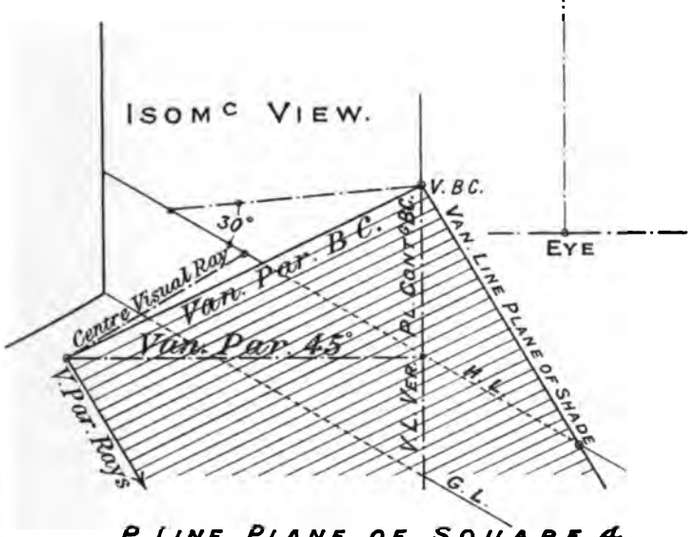
PROJECTION OF SHADOWS

RAYS PARALLEL TO PICTURE PLANE.

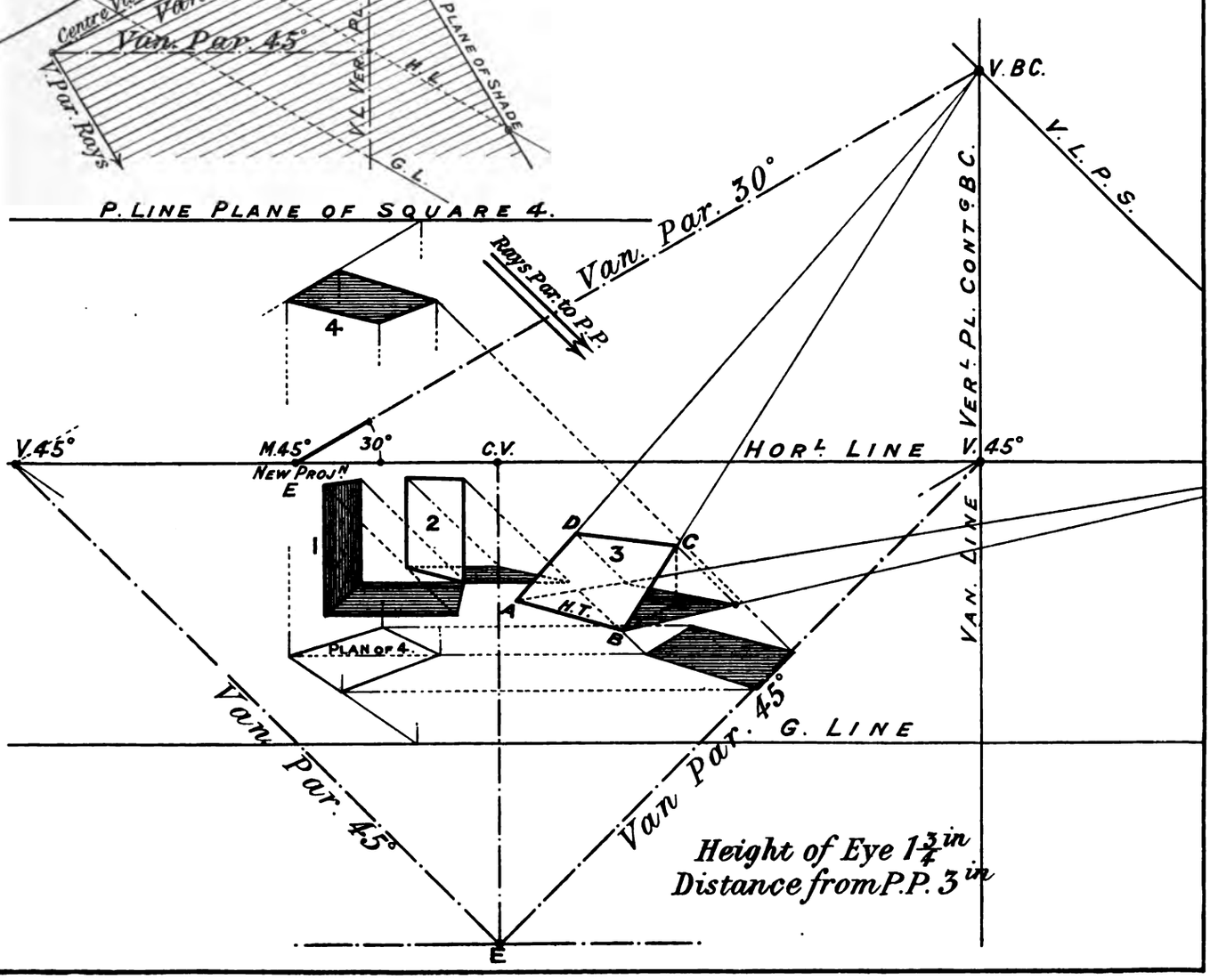


PLAN OF PICTURE PLANE

Scale $\frac{1}{2}$



P. LINE PLANE OF SQUARE 4.



Height of Eye $1\frac{3}{4}$ in
Distance from P.P. 3 in

PLATE XIX.

SHADOW OF OBLIQUE SQUARE WHEN SUN IS BEHIND SPECTATOR.

The plane of a 3″ square is oblique, its inclination to the horizon being 30°, and its H.T. vanishing 30° to the left. Near corner of under edge 2″ to the left and 1″ within.

The horizontal or ground plane it rests on is $2\frac{1}{2}$″ below the level of the eye.

To find its shadow when the sun is behind the spectator's left, and at an altitude of 45°, the rays descending in parallel planes, which make **45°** with the **P.P.** towards the right.

In accordance with the rule laid down in the previous examples, and after having obtained the **V.P.** for the rays, **V.P.S.R.**, to find shadow of sloping edge AC. Join **V.P.S.R.** to **V.P.** for AC, giving the **V.L.** for the plane of shade and its intersection with the **H.L.** (the **V.L.** of the plane receiving the shadow). This intersection will be the **V.P.** for the shadow-line, and its length will then be determined by a ray through C. Shadow of CD will vanish to **V.P.** CD.

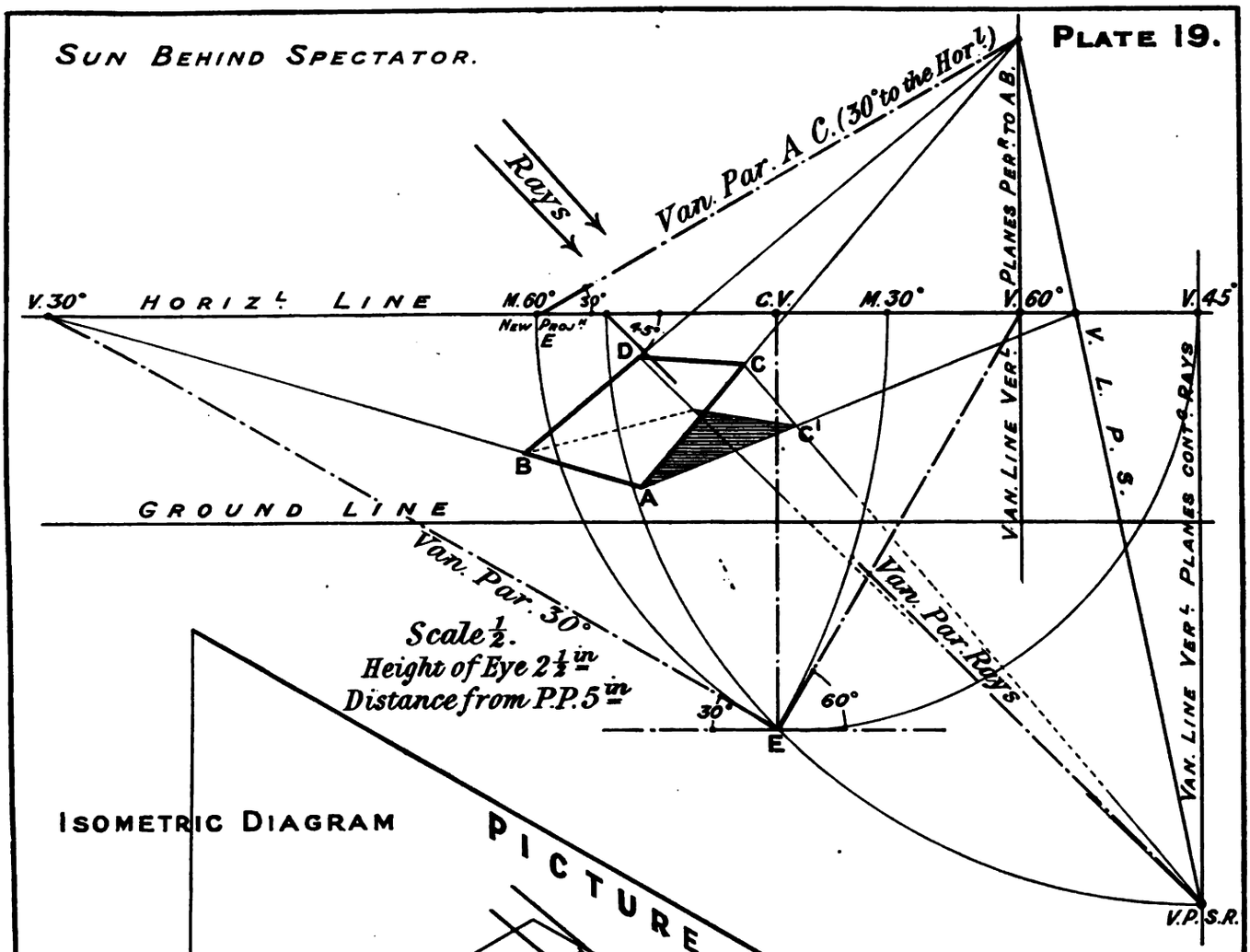
All this, which seems simple enough in the perspective diagram, is best explained by use of an isometric drawing.

This diagram will show that the problem consists in carrying on operations in *front* of the **P.P.** (between the eye and the **P.P.**) in accordance with the known position of lines on the other side, or *within* the **P.P.**

The vanishing parallel line for edge AC—that is, the line parallel to AC passing from **E.** to meet the **P.P.**—gives **V.P.** AC. A ray is drawn through C; and to determine where it meets the **G.P.**, or to find AC′, we draw through **E.** a vanishing parallel to the rays meeting the **P.P.** in **V.P.S.R.** Joining **V.** AC and **V.P.S.R.**, we obtain the intersection with the **P.P.** of a plane containing vanishing parallel AC and vanishing parallel rays, which is thus a parallel plane to ACC′. The horizontal plane through **E.**, which meets the **P.P.** in **H.L.**, and which is the vanishing parallel of the **G.P.** receiving the shadow, is intersected by this plane in the line "vanishing parallel shadow of AC," and so is parallel to the line AC′; and therefore where it crosses the **H.L.** will be the **V.P.** for AC′ or shadow of AC.

SUN BEHIND SPECTATOR.

PLATE 19.



ISOMETRIC DIAGRAM

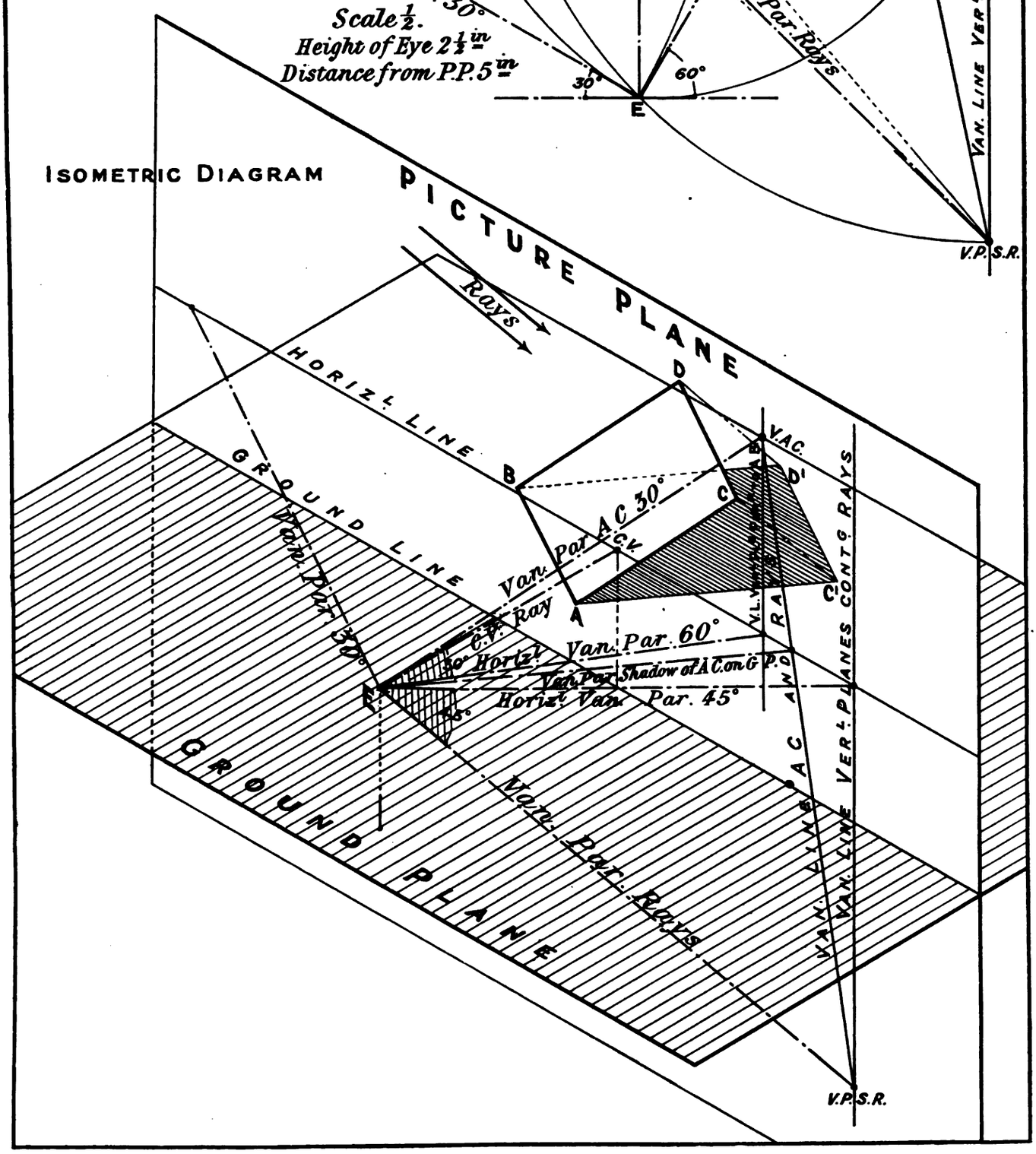


PLATE XX.

SQUARE FRAME RESTING AGAINST A PRISM—SUN BEHIND THE SPECTATOR.

A Frame 5″ square, material 1″ square, rests against a square prism 9″ long with ends 2·8″ square.

Near face $4\frac{1}{2}$″ within and parallel to the P.P., nearest corner of base 3″ to the left.

The frame rests against front of prism, not centrally, but 1″ to the left of its centre.

Its under face is inclined 45°—that is, it is in an ascending plane inclined 45°.

Sun behind the spectator's left, altitude 25°, rays in parallel planes inclined 35° to the P.P. towards the right.

Height of eye $4\frac{1}{2}$″. Distance from **P.P.** 8″. Scale $\frac{1}{2}$.

Beginning with prism, obtain its under face, nearest corner of which is 3″ to the left and $4\frac{1}{2}$″ within. As regards the frame, we may either begin with the **H.T.** of the under face, which will be at a distance of 2·8″ in front of the prism (equal to height of prism): that is, its **H.T.** will be 1·7″ within, one end of it being 1″ to the left; or we may start from top edge of prism, where the frame touches 1″ from its left end, and work downwards till we reach the **G.P.**

To complete the perspective of the frame, suppose we have obtained the under edge resting on the **G.P.**, which is 5″ long, then, to determine **V.L.** for ascending planes of top and bottom faces, and descending plane for under visible edge, we project **E.** over into the **H.L.** (as has been already explained) and set off above and below the **H.L.** the vanishing parallels **45**″; and from these obtain the **V.L.'s** for the planes of the frame.

On these, again, **V.P.'s** on right and left for lines in them inclined **45°** are found. Having marked off 1″, thickness of material at each end of edge, which is on the **H.T.**, and joined these to the **V.P.'s** above and below **C.V.** (the **V.P.'s** for lines perpendicular to the **H.T.** of the ascending and descending planes), these lines will then represent widths of material of the sides. To measure their lengths upwards, on the descending plane of the visible edge of frame, we draw their diagonals to **V.P.'s 45°**, and so obtain under edge of upper face. These, again, taken to **V.P.** directly over **C.V.**, and diagonals drawn to **V.P.'s 45°** in ascending plane, will cut off length of upper face.

For the shadows—after finishing that of the square prism—seen on the right, we begin with that of edge BC of frame. **V.P.S.R.** joined to **V.P.** BC will cross **H.L.** at **V.P.** required for shadow on **G.P.**, and to draw it we must either produce BC to meet the **G.P.**—find its **H.T.** (as has been done here), and with a ray through B determine the beginning of the shadow line; or, commencing at corner A, which is on the **G.P.**, find shadow of AB, assuming that AB does throw a shadow (is not itself in shade) we find shadow of B, and then proceed with that of BC.

V.P. for shadow of AB is considerably beyond the limits of our paper, as will be evident, and may be dispensed with—though a perpendicular dropped from B to the **G.P.**, and *its* shadow could be utilized in order to find that of B.

Following up shadow of BC on the **G.P.**, we find it runs into the shadow of the upright edge of prism, and so we must trace back from this point (the intersection of the shadow lines) *along a ray*—that is, from **V.P.S.R.**—a line to meet the upright edge of prism; and so continue the remainder of the shadow of BC on the front, and afterwards on the top face of the prism.

Because the front face of the prism is parallel to the **P.P.**, the shadow of edge BC on it will be a line parallel to **V.L.P.S.** for BC; and since by drawing a ray through C we find the shadow is continued on upper face of prism, it will be completed by a line to the **V.P.** on the **H.L.**, to which the shadow on the **G.P.** was drawn.

Upper horizontal edge through C throws a shadow parallel to itself.

For shadow of left-hand side of frame we know by inspection that the long edge through D, and the edge diagonally opposite, will determine the width of the shadow; and so we treat them as we do edge BC.

The inner surface at top side of frame receives a shadow, the **V.P.** for which must be in the **V.L.** of the plane receiving the shadow, *i.e.* that of the descending plane, and so it has been found here.

PLATE 20.

SUN BEHIND SPECTATOR

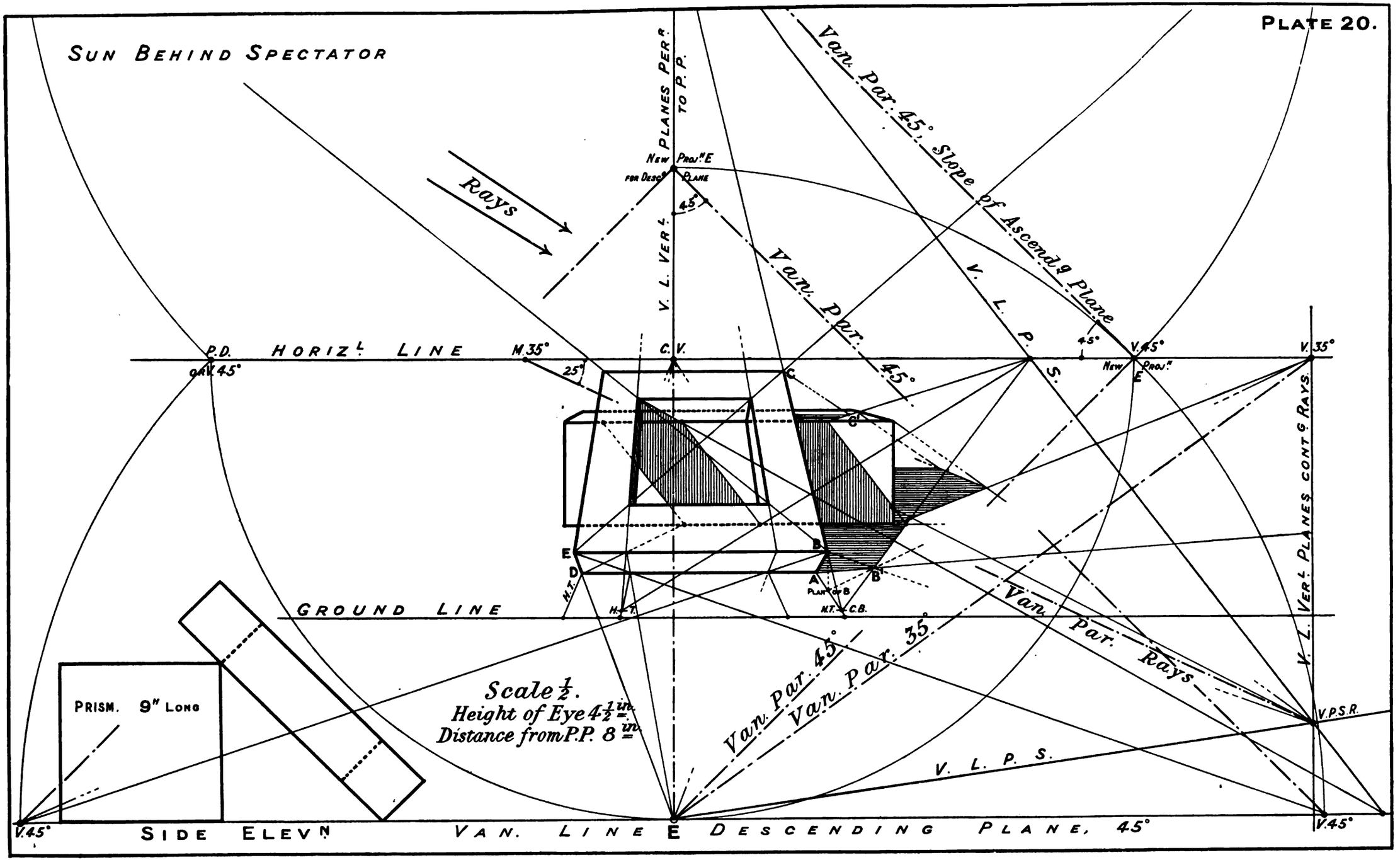


PLATE XXI.

SHADOW OF SQUARE FRAME ON PYRAMID.

A Frame 4″ square, material $\frac{1}{2}$″ square. Stands upright, nearest vertical edge directly in front and 2″ within. Faces of frame vanish 30° to the left.

Edges of Pyramid base, which is $2\frac{1}{2}$″ square, are equally inclined to the P.P. nearest corner $1\frac{1}{2}$″ to the right and $3\frac{1}{2}$″ within.

Altitude of Pyramid $3\frac{1}{2}$″.

The horizontal plane on which these models rest is 4″ below the eye, and the eye is 7″ from the P.P. Scale $\frac{1}{2}$.

Sun behind spectator's left, altitude 30°. Rays in parallel planes inclined 35° to the P.P.

To obtain shadows we had better begin by finding that of the Pyramid, which resolves itself into finding shadow of vertex considered as the top of a vertical line (the axis), and joining whichever corners of the Pyramid base are outermost.

This will at once show which faces of the Pyramid are in light; and therefore in this case will receive part of shadow of frame.

Shadow of frame then found on the **G.P.**, and in order to do so produce vertical edges which are seen to throw shadows down to the **G.P.**, and start their shadows from these points —their **H.T.'s**.

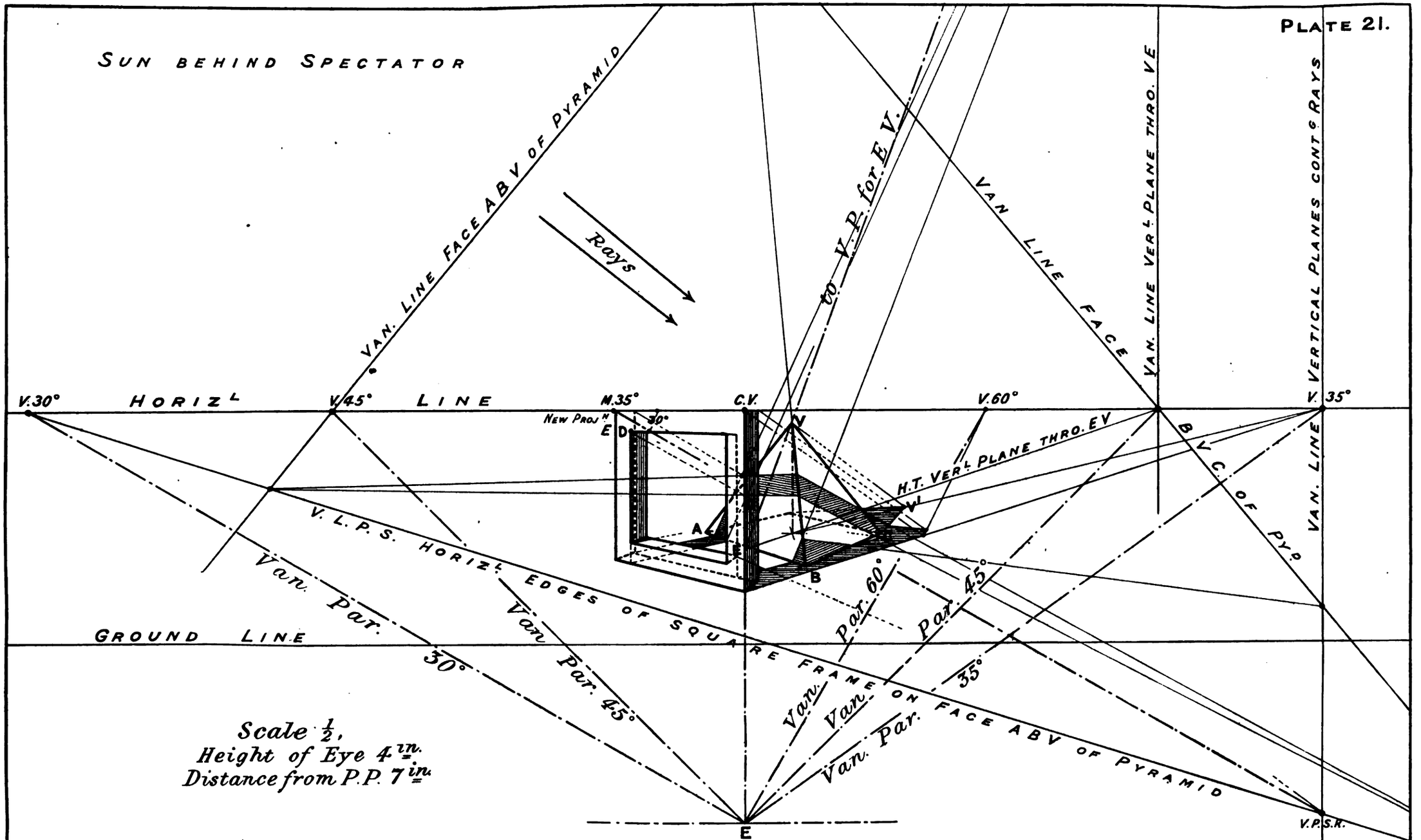
Notice also shadow on top of lower bar. To continue shadows on Pyramid faces we require their **V.L.'s**.

Beginning with face ABV, assume a vertical plane bisecting this face and containing the axis—that is a plane perpendicular to edge AC, and hence vanishing **45°** to the right. The **V.L.** of this plane will contain the **V.P.** for EV, and so, knowing the **V.P.'s** for two lines in the plane, we can determine the **V.L.** for the face ABV. **V.P.** for edge BV, common to faces ABV and BVC is next found on **V.L.** face ABV, and joined to **V.P.** BC, will give **V.L.** for face BVC. As the shadow of the far upright bar reaches AB—base edge of Pyramid—we next continue it on that face by finding **V.P.** for it. That is, seeing the line throwing the shadow is vertical, a vertical line through **V.P.S.R.**, to cut **V.L.** face ABV, will give it (beyond the paper however).

The shadow of the near edge of the bar we are dealing with terminates when we meet the under line of the top horizontal bar at D, and a ray will define the shadow of D.

V.P. for horizontal line shadow next is found as usual: **V.P. 30°** on left to **V.P.S.R.** cuts **V.L.** ABV in **V.P.** required.

Back upper edge terminates also when it meets upper line of horizontal bar, and, if necessary, assume face of Pyramid produced to receive it. The **V.P.** for this shadow line we have just found: both are brought over to edge BV separating the two Pyramid faces; and to continue shadow lines on this face seek for their **V.P.'s**, both for that of top and for a portion of the vertical edge of near bar, which also throws shadow in Pyramid corner.



Scale $\frac{1}{2}$,
Height of Eye 4 in.
Distance from P.P. 7 in.

PLATE XXII.

SHADOWS OF CYLINDER AND CONE ON GROUND PLANE—SUN IN FRONT OF SPECTATOR.

A Cylinder 7″ long, $3\frac{1}{2}$″ diameter, lies on a horizontal or ground plane. Axis vanishing 60° to the left. Near end of it 2″ to the right and 5″ within.

A Cone 3″ diameter of base, altitude 5″. Stands upright. Axis $4\frac{1}{2}$″ to the right and $4\frac{1}{2}$″ within.

Sun is in front of spectator's right. Altitude 35°; rays in parallel planes making 60° with the P.P.

As regards cone, shadow of axis will determine that of its vertex—brought in front of **P.P.**, if necessary, as here—and from it tangents to the base will enclose the shadow.

If it be required to mark, on the surface of the cone, the visible line separating light and shade, the line, in fact, whose shadow is V'A, we would require to draw a line from centre of base perpendicular to the tangent V'A. This is done by finding **V.P.** for V'A, and joining to **E.**, thus obtaining its vanishing parallel, and setting off another vanishing parallel perpendicular to it. This, then, gives **V.P.** in the **H.L.** for line CA.

Shadow of cylinder resolves itself into that of the ends (ellipses in perspective) and tangents to them. We can find, however, first, what portion of cylinder end will throw a shadow. To do so obtain **V.L.** of plane containing the sun and the axis of the cylinder, and produce to cut **V.L.** of vertical plane of cylinder end.

Tangent lines from this intersection to cylinder end (we are dealing with near end at present) will separate light and shade. Upper tangent through 5 produced to meet **H.T.** of plane of cylinder end will give the point where the straight shadow-line crosses this **H.T.**

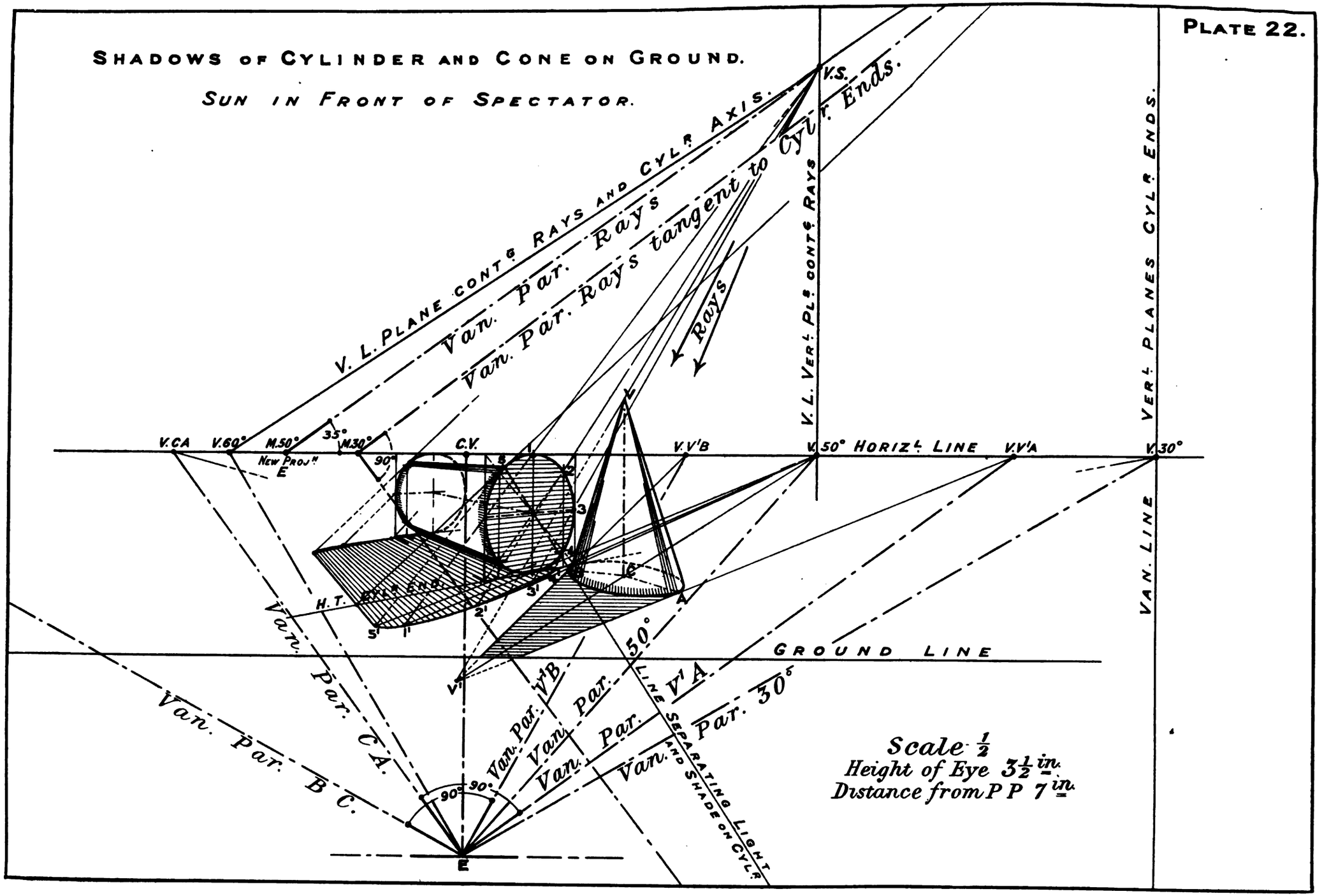
This does not limit the length of the shadow though : it is brought forward to meet a ray from **V.S.** through 5, and then this straight line of the shadow will vanish to the **V.P.** of the axis. To define exactly where these tangents touch the ellipse—that is, points 5 and 1—we require to draw through centre of cylinder end a line perpendicular to these tangents. Draw the vanishing parallel of the tangent, and a perpendicular to it brought to meet **V.L.** of cylinder end will give its **V.P.**

To find shadow of intermediate points between 4 and 5, drop a perpendicular from each point, and in every case, by finding shadows of these vertical lines, we obtain that of the points selected as 1, 2, and 3.

PLATE 22.

SHADOWS OF CYLINDER AND CONE ON GROUND.

SUN IN FRONT OF SPECTATOR.



Scale $\frac{1}{2}$
Height of Eye $3\frac{1}{2}$ in.
Distance from P P 7 in.

PLATE XXII.

SHADOWS OF CYLINDER AND CONE ON GROUND PLANE—SUN IN FRONT OF SPECTATOR.

A Cylinder 7″ long, $3\frac{1}{2}$″ diameter, lies on a horizontal or ground plane. Axis vanishing 60° to the left. Near end of it 2″ to the right and 5″ within.

A Cone 3″ diameter of base, altitude 5″. Stands upright. Axis $4\frac{1}{2}$″ to the right and $4\frac{1}{2}$″ within.

Sun is in front of spectator's right. Altitude 35°; rays in parallel planes making 50° with the P.P.

As regards cone, shadow of axis will determine that of its vertex—brought in front of **P.P.**, if necessary, as here—and from it tangents to the base will enclose the shadow.

If it be required to mark, on the surface of the cone, the visible line separating light and shade, the line, in fact, whose shadow is V'A, we would require to draw a line from centre of base perpendicular to the tangent V'A. This is done by finding **V.P.** for V'A, and joining to **E.**, thus obtaining its vanishing parallel, and setting off another vanishing parallel perpendicular to it. This, then, gives **V.P.** in the **H.L.** for line CA.

Shadow of cylinder resolves itself into that of the ends (ellipses in perspective) and tangents to them. We can find, however, first, what portion of cylinder end will throw a shadow. To do so obtain **V.L.** of plane containing the sun and the axis of the cylinder, and produce to cut **V.L.** of vertical plane of cylinder end.

Tangent lines from this intersection to cylinder end (we are dealing with near end at present) will separate light and shade. Upper tangent through 5 produced to meet **H.T.** of plane of cylinder end will give the point where the straight shadow-line crosses this **H.T.**

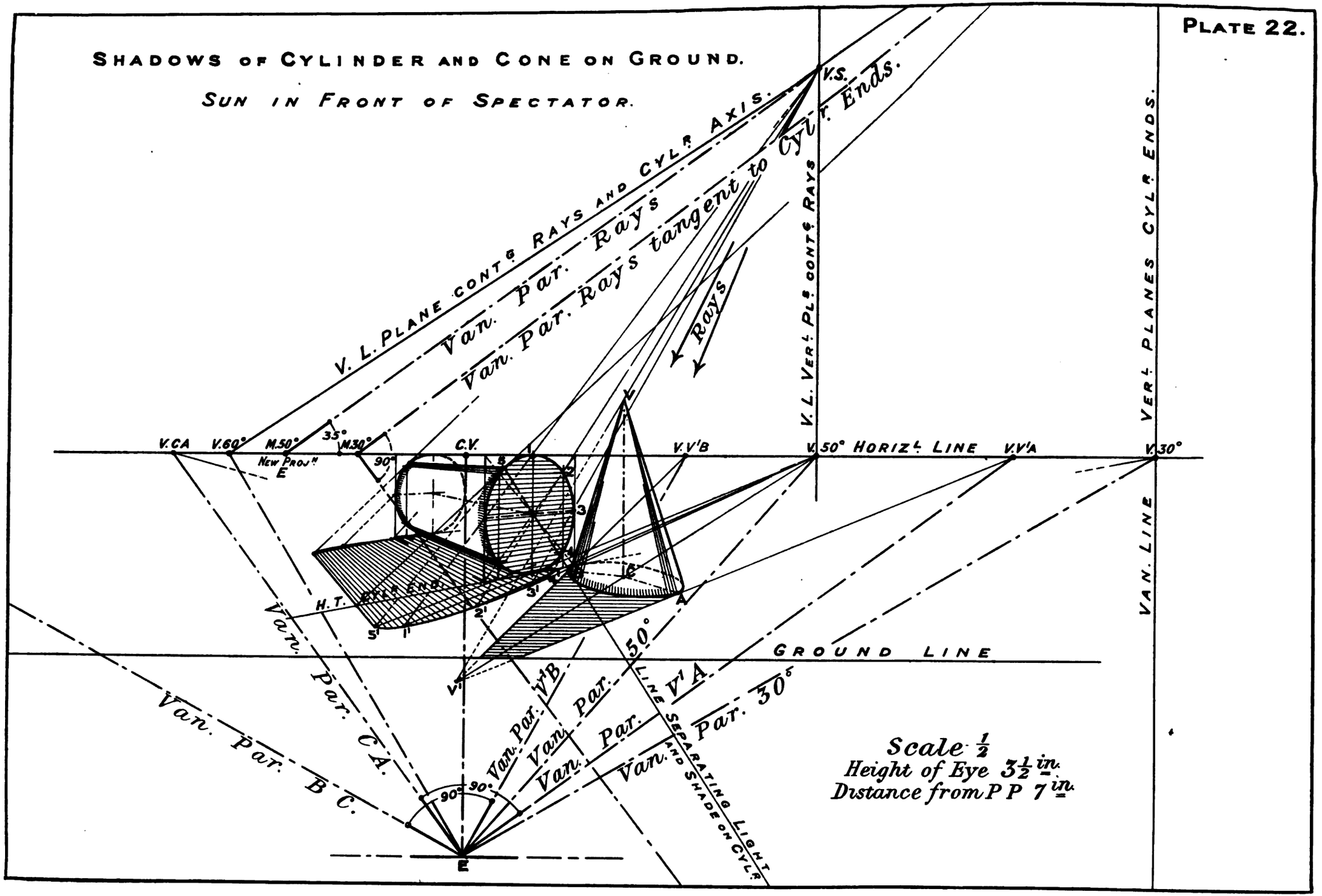
This does not limit the length of the shadow though : it is brought forward to meet a ray from **V.S.** through 5, and then this straight line of the shadow will vanish to the **V.P.** of the axis. To define exactly where these tangents touch the ellipse—that is, points 5 and 1—we require to draw through centre of cylinder end a line perpendicular to these tangents. Draw the vanishing parallel of the tangent, and a perpendicular to it brought to meet **V.L.** of cylinder end will give its **V.P.**

To find shadow of intermediate points between 4 and 5, drop a perpendicular from each point, and in every case, by finding shadows of these vertical lines, we obtain that of the points selected as 1, 2, and 3.

PLATE 22.

SHADOWS OF CYLINDER AND CONE ON GROUND.

SUN IN FRONT OF SPECTATOR.



Scale $\frac{1}{2}$
Height of Eye $3\frac{1}{2}$ in.
Distance from P P 7 in.

PLATE XXIII.

SHADOW OF PRISM ON CYLINDER—SUN BEHIND SPECTATOR.

A Cylinder lies on a horizontal or ground plane, which is $4\frac{1}{2}$″ below the eye. Its axis vanishes 30° to the left. Near end of it 2″ to the right and 6″ within. Diameter of cylinder 4″, length 7″.

Square prism rests against middle of length of cylinder, under face inclined 45°; prism is 8″ long and 2″ square.

Sun is behind the spectator's left at 35° altitude. Rays in parallel planes inclined 35° to P.P.

It is assumed that the student requires no help in obtaining the perspective representation of the two models, and so we proceed with finding their shadows.

That of the cylinder—as much as is visible—will be found as in the last example.

The **V.L.** for cylinder axis and rays is obtained, and where it crosses **V.L.** for cylinder end is **V.P.** to which tangents are drawn to ellipse to cut **H.T.** of vertical plane of cylinder end. These give points in the straight lines bounding the shadow, though not the terminals of these lines, which are found by rays to **V.P.S.R.**

Shadows of points 2, 3, and 4 next found as before explained.

Obtain complete shadow also of prism on the **G.P.**; and then we proceed to find shadow of prism on cylinder. Through the points on the cylinder end which were selected for obtaining shadow on **G.P.**—through 2, 3, and 4—draw straight lines on cylinder surface, that is, draw lines to **V.P.** for axis. Then similar lines through the shadows of points 2, 3, and 4 will be the shadows of the corresponding lines on the cylinder surface.

Select new edges of prism which evidently throw shadows on the cylinder, and seek for the *intersections of their shadows with those of lines* 2, 3, and 4. That is, from where shadow of edge AB crosses shadows of lines through 2, 3, 4, trace back, by lines from **V.P.S.R.**, to where the corresponding lines are on the cylinder.

These, then, are points in the shadow of the prism on the cylinder.

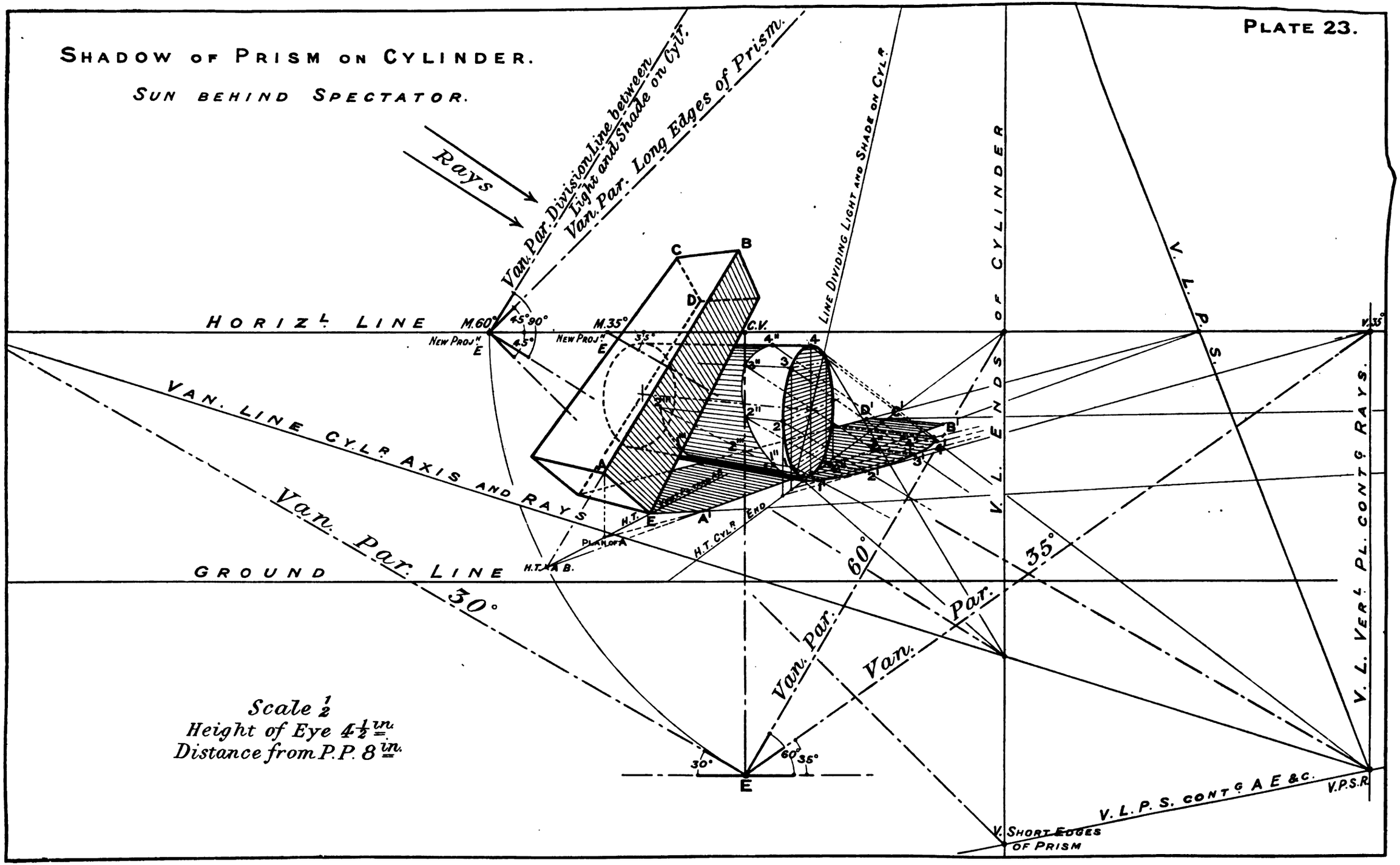
For it is evident that, assuming these points in the shadow of AB on the cylinder to be correct, then the transference of these lines selected on the cylinder, and that of edge AB on to the **G.P.**, as shadows will not disturb their mutual relation to the rays of light.

The ray through 3‴ on AB will meet cylinder at 3″ and pass on to the **G.P.** at 3″, a point common to shadow of AB and of line through 3 on cylinder. This method of finding shadows on the surface of any intervening body may be utilized frequently to advantage.

PLATE 23.

SHADOW OF PRISM ON CYLINDER.

SUN BEHIND SPECTATOR.



Scale $\frac{1}{2}$
Height of Eye $4\frac{1}{2}$ *in.*
Distance from P.P. 8 *in.*

PLATE XXIV.

PORTION OF A ROOF WITH CHIMNEY STALK COMING THROUGH, SHOWN IN PERSPECTIVE, TO PROJECT SHADOWS WHEN THE SUN IS IN FRONT OF THE SPECTATOR (BEHIND THE P.P.) ON THE LEFT, ALTITUDE 45°. RAYS LYING IN VERTICAL PLANES INCLINED 30° TO THE P.P.

Assuming that such a stretch of roofing as is shown (repeated to $\frac{1}{2}$ scale clear of construction lines) is given, together with the position of the **C.V.**, to project the resultant shadows when the source of light is as indicated. In order to determine **V.P.** for rays we must find projection of the eye on to the **P.P.**

We do this by producing lines of roof which are evidently at right angles to their **V.P.'s** on the **H.L.** through **C.V.** FD to right and ED to the left. Describing a semicircle on the **H.L.** with these **V.P.'s** as extremities of the diameter will cut a perpendicular through **C.V.** in **E.**

Set off in the usual way vanishing parallel for the sun: that is, **30°** to the left, with Line of Direction, will give the **V.L.** for planes containing the rays, and **45°** at the new projection of **E.** in the **H.L.** will give **V.P.** for the sun.

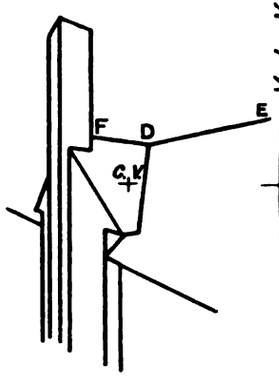
Beginning with shadow of chimney, nearest face is evidently in shade: vertical edge through C, therefore, will throw shadow, and also that through A. Produce roof surface across chimney till it cut front edge at G, and commence shadow here; and having obtained **V.L.** for roof slope containing DF—the surface here receiving the shadow—we project downward from **V.** sun to cut **V.L.** of roof plane—because line throwing shadow is vertical—thus obtaining **V.P.** for shadows of edges through C and A. Shadow is then continued over adjoining roof plane, that through DE, and **V.P.** for shadows obtained as before on the **V.L.** of that plane.

Complete shadow by getting those of edges of chimney-top AB and BC.

To find shadow of upright edge through H we find it first on the supposed horizontal surface under the eaves, and then a vertical will bring it into sight.

PLATE 24.

SUN IN FRONT OF SPECTATOR.



SKETCH VIEW
Scale ½.

S
V.L. VERL. PLANES CONTG RAYS
Van. Par. Rays
Rays
V. L. P. S.
VAN. LINE
OF ROOF DE
SLOPE
SLOPE OF ROOF DF
E
C
B
A
F
D
G
A'
B'
C'
45°
VAN. LINE FACE OF WALL
V.30° HORIZL
V.DE
VAN. LINE
V.DF
VAN. LINE PERPR. TO FACE OF WALL
H
H'
Van. Par.
30°
Van. Par. DF
30°
E

PLATE XXV.

RADIAL PROJECTION OF SHADOWS—SOURCE OF LIGHT A POINT.

In examples of artificial lighting, where the source of light is a point, usually in close proximity to the material affected by the light, we have to deal with radial projection, where the resultant shadow will, in many cases, be considerably larger than the object causing it.

Again, however, as in our previous examples of parallel rays, the shadow of any line will be the intersection of a plane containing the line throwing the shadow and the source of light, **L.**, with the plane receiving the shadow. Translated into perspective phraseology, the **V.L.** of the shadow-line will be the intersection of the **V.L.P.S.** and the **V.L.** of the receiving plane. Now, in order to obtain this **V.L.P.S.**—that is the **V.L.** for a plane containing a line and a point—we must select some suitable point in the line and join **L.** to it; and so the problem simplifies itself into finding the **V.P.'s** for two lines which meet.

The corner of a room is shown in Perspective; a portion of a Table, and a Wall-cupboard (shown in block form) supported by a bracket below its centre. In left-hand wall a door-opening, the door being thrown open (probably the table would require removal in order that the door may shut, but that need not affect the problem here !).

On top of the table is placed a light, L., the position of which is fixed by its plan on the table-top being indicated.

The problem is to determine the various shadows resulting from this combination. In this example the problem can be worked out without previously obtaining the **V.L.'s** for the walls. The **H.L.** can, however, be drawn through centre of the circle, which is the limit of the Field of Vision. Having given L_1, plan of L on table-top, we can now obtain its horizontal projection, or plan, on the floor. We do this by seeking for the plan of *any line* which lies on the table-top. Now, the only lines we can identify above and below are those connected with the legs. Join L_1 to any point connected with the ends of the square legs which are visible, and reproduce the line below by the use of its **V.P.** We thus obtain L_{11}, and then as regards shadow of table obtain plan on floor of one corner A. **V.P.** L_1A being found, $L_{11}A_1$ will vanish to same point.

A ray LA produced to meet $L_{11}A_1$ will determine shadow of corner because $L_{11}A_1$ is **H.T.** of a vertical plane containing L, and the point throwing the shadow.

Edges of table throw shadows vanishing parallel to themselves, being horizontal. As for cupboard, the **H.T.'s** of vertical planes containing the upright edges and **L.** are found, and where they meet the **H.T.** of the wall verticals are drawn which contain their shadows, and their lengths are limited by rays from **L.** To obtain these **H.T.'s** just mentioned, the plan of the cupboard on the floor must be found. From the position of the plan of the supporting bracket with reference to L_{11} the visible portion is evidently in light; and so from where shadow of under edge of bracket meets it a line is drawn across to top of front, because front edge is *flush* with cupboard front.

Shadow of door obtained in the same way, as also side of doorway on wall in the adjoining room.

PLATE 25.

RADIAL PROJECTION OF SHADOWS.

SOURCE OF LIGHT A POINT.

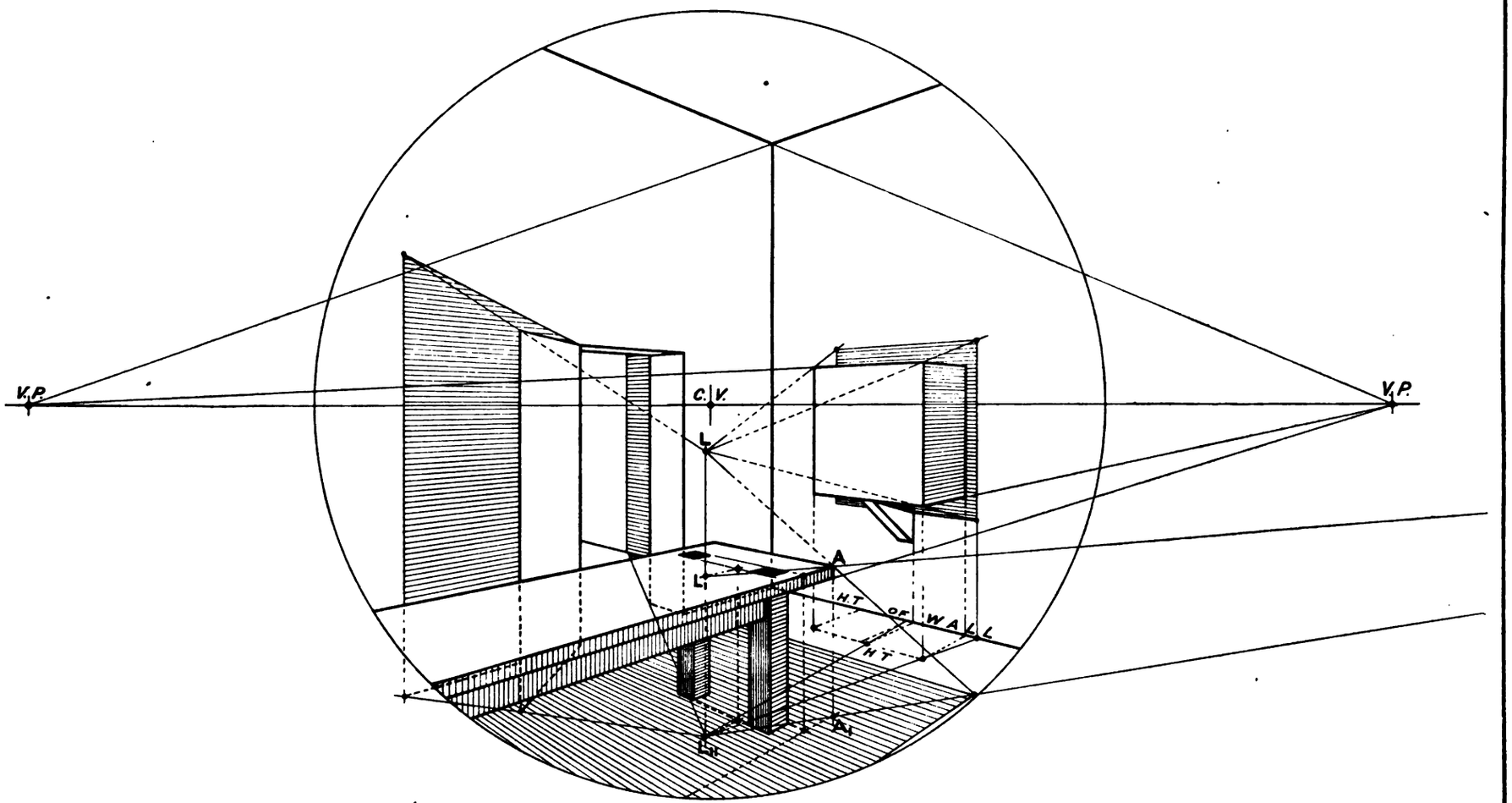


PLATE XXVI.

THE PERSPECTIVE OF A HOODED DOORWAY AND THE POSITION OF L., AN ARTIFICIAL LIGHT, BEING GIVEN TO DETERMINE THE SHADOWS.

In this example, owing to a majority of the lines being oblique, we require to find the **V.L.'s** of the various planes.

We begin by finding plan of the hood, and because **L.** is not opposite middle of doorway, we see the shadow of the hood on the wall will not be symmetrical.

To obtain shadows of BA and DE, upright edges, we find **H.T.'s** of vertical planes containing them and **L.**, and from their intersections with the **H.T.** of the wall the uprights containing their shadows. Their lengths are then limited by rays from **L.**

As for those of the horizontal under edges A and E, they will form parts of the same straight line, vanishing to the horizontal of the plane of the wall. By projecting **L.** on to the wall, at L', we see that the sloping upper face of the hood on the right—the plane MCD—is in shade, because L' falls below the sloping line through M, when continued. Edges MC and CD will, therefore, throw shadows, though only a portion of that of CD will be visible.

As regards the under side of hood and the shadow of edge FG on it, we had better find **V.L.** for plane FLG. That is, the **V.L.** for the vertical plane through LG is found containing **V.P.** for line LG in it; and this joined to **V.P.** FG will give **V.L.P.S.** required, the intersection of this with that marked **V.L.** Plane MCD (a parallel plane to the one receiving the shadow) will give **V.P.** for shadow of GF. On reaching wall surface, which is now parallel to line throwing shadow, the remainder of shadow will vanish to same **V.P.**

Lintel of doorway will throw shadow on jamb, of which the bevelled or chamfered portion is parallel to the **P.P.**

The **V.L.P.S.** for this is found—that is, **V.P.** for LH (first finding **V.L.** of vertical plane through LH) joined to **V.P.** KH, and a parallel to it will give the required shadow, because the plane here receiving it is parallel to the **P.P.**

As regards the other visible portion of the jamb—that opposite spectator's left—the continuation of the shadow of FG, unfinished on the wall above, will be found on it.

By continuing it across, we cut the upright edge through K (and a small portion between this and the bottom of the hood may be seen also), and to obtain **V.P.** for shadow on the side of the door—which vanishes through **C.V.**—find intersection of **V.L.** Plane FLG with the perpendicular through **C.V.**

PLATE 26.

DOORWAY WITH PROJECTING HOOD. *ARTIFICIAL LIGHT.*

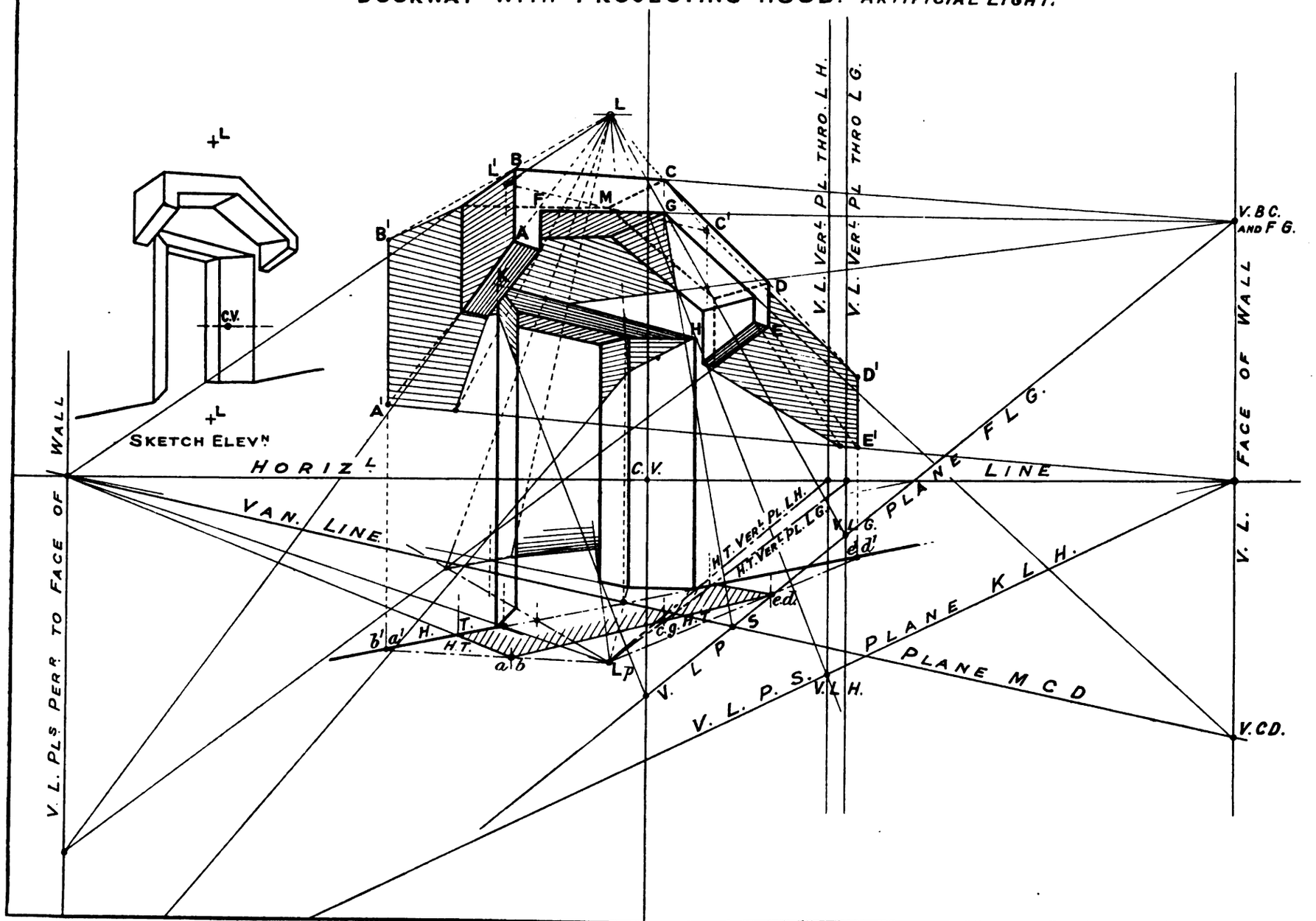


PLATE XXVI.

THE PERSPECTIVE OF A HOODED DOORWAY AND THE POSITION OF L., AN ARTIFICIAL LIGHT, BEING GIVEN TO DETERMINE THE SHADOWS.

In this example, owing to a majority of the lines being oblique, we require to find the **V.L.'s** of the various planes.

We begin by finding plan of the hood, and because **L.** is not opposite middle of doorway, we see the shadow of the hood on the wall will not be symmetrical.

To obtain shadows of BA and DE, upright edges, we find **H.T.'s** of vertical planes containing them and **L.**, and from their intersections with the **H.T.** of the wall the uprights containing their shadows. Their lengths are then limited by rays from **L.**

As for those of the horizontal under edges A and E, they will form parts of the same straight line, vanishing to the horizontal of the plane of the wall. By projecting **L.** on to the wall, at L', we see that the sloping upper face of the hood on the right—the plane MCD—is in shade, because L' falls below the sloping line through M, when continued. Edges MC and CD will, therefore, throw shadows, though only a portion of that of CD will be visible.

As regards the under side of hood and the shadow of edge FG on it, we had better find **V.L.** for plane FLG. That is, the **V.L.** for the vertical plane through LG is found containing **V.P.** for line LG in it; and this joined to **V.P.** FG will give **V.L.P.S.** required, the intersection of this with that marked **V.L.** Plane MCD (a parallel plane to the one receiving the shadow) will give **V.P.** for shadow of GF. On reaching wall surface, which is now parallel to line throwing shadow, the remainder of shadow will vanish to same **V.P.**

Lintel of doorway will throw shadow on jamb, of which the bevelled or chamfered portion is parallel to the **P.P.**

The **V.L.P.S.** for this is found—that is, **V.P.** for LH (first finding **V.L.** of vertical plane through LH) joined to **V.P.** KH, and a parallel to it will give the required shadow, because the plane here receiving it is parallel to the **P.P.**

As regards the other visible portion of the jamb—that opposite spectator's left—the continuation of the shadow of FG, unfinished on the wall above, will be found on it.

By continuing it across, we cut the upright edge through K (and a small portion between this and the bottom of the hood may be seen also), and to obtain **V.P.** for shadow on the side of the door—which vanishes through **C.V.**—find intersection of **V.L.** Plane FLG with the perpendicular through **C.V.**

PLATE 26.

DOORWAY WITH PROJECTING HOOD. *ARTIFICIAL LIGHT.*

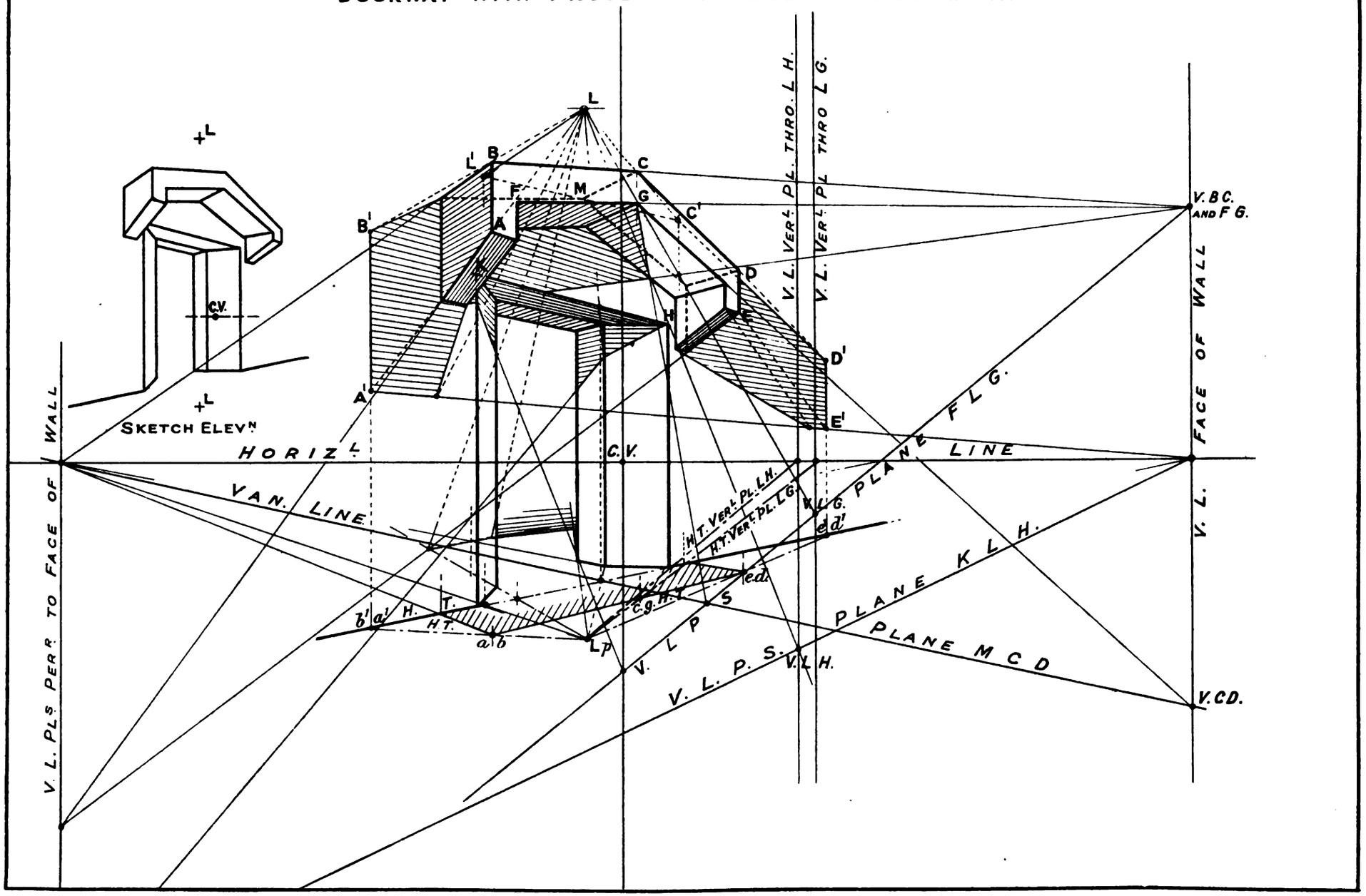


PLATE XXVII.

REFLECTIONS IN VERTICAL MIRRORS.

When light falls on a plane surface capable of reflecting it in a definite manner, the incident and reflected rays lie in the same plane with the normal (the perpendicular to the surface) at the point of incidence, and are on opposite sides making equal angles with it; the original ray—the one striking the reflecting surface—being called the incident ray.

In Fig. 1.—Let XY be the plan of a vertical mirror, a perfectly reflecting surface, and assuming that **E. P.** and (which may or may not be in the same horizontal plane) are given—**E.** representing the eye and **P.** the object, the reflection of which is sought for in the mirror. Draw through **P.** a perpendicular to the plane of the mirror, and from I a point in this perpendicular at the same distance behind that **P.** is in front, draw a line to **E.** intersecting the plane of the mirror at R; then from the similarity of triangles in the diagram, the angle PRN equal angle ERN; *i.e.* the angle of incidence being equal to the angle of reflection, a ray of light from **P.** to meet the eye will be reflected from the mirror at **R.**

And as the reflected ray *seems to come* from I, this point is termed the *optical* or *virtual* image of P.

Fig. 2 shows, in plan and elevation, a cube of $1\frac{1}{2}$″ edge resting on a horizontal or ground-plane 3″ below the eye. The front face of the cube is parallel to and 2″ from the P.P., nearest corner $\frac{1}{4}$″ to the right.

$2\frac{5}{8}$″ to the right of the centre stands a vertical mirror perpendicular to the P.P. And to the left, meeting the P.P. $2\frac{1}{4}$″ from the centre, is another vertical mirror which is inclined 60° to the P.P. towards the right.

The problem is to draw the Perspective representation of the cube together with its reflections in the two given mirrors.

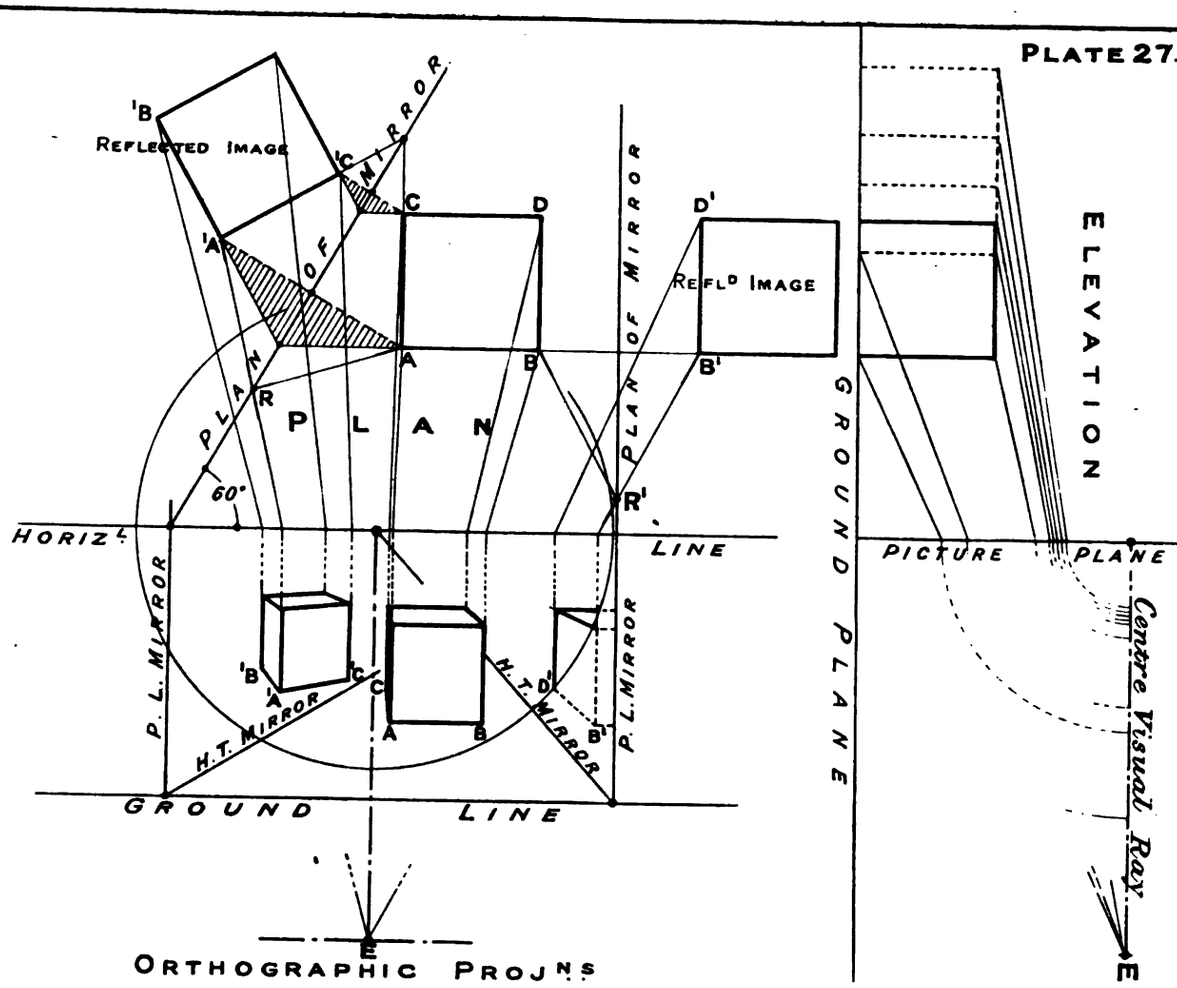
The cube, together with the position of the optical images required for obtaining the reflections, are shown; and the resultant is obtained by simple horizontal and vertical projection. This, of course, has merely been done in order to impress on the student the necessity of *imagining* the locality of the *image* he is in search of.

Referring now to the perspective diagram, because the right-hand mirror is perpendicular to the **P.P.**, and the faces of the cube are parallel and perpendicular to the **P.P.**, the reflected image will be found by continuing edge AB to meet the **H.T.** of the mirror, and producing it as far to right of mirror as B is to left, joining to **C.V.**, and bringing across back edge of cube to cut off visible length of side edge. Vertical edges are next drawn, which are then cut by horizontals from the original cube; and selecting from all this as much as lies within the " Field of Vision."

As regards left-hand mirror, we arrive at the image of AC by a perpendicular through A to the plane of the mirror, and measuring ^{1}A as far beyond as A is in front. Or by similar triangles we reach ^{1}A by producing BA to meet **H.T.** of mirror and setting off, beyond this point, a line at the same angle with the mirror plane which BA makes with it. **V.P.'s** for all these are obtained at E—that is, a vanishing parallel for lines perpendicular to mirror for A ^{1}A—and, because AB is parallel to the **P.P.** and plane of the mirror makes **60°** with it to the right, **60°** on the other side will give vanishing parallel for reflected image ^{1}A ^{1}B.

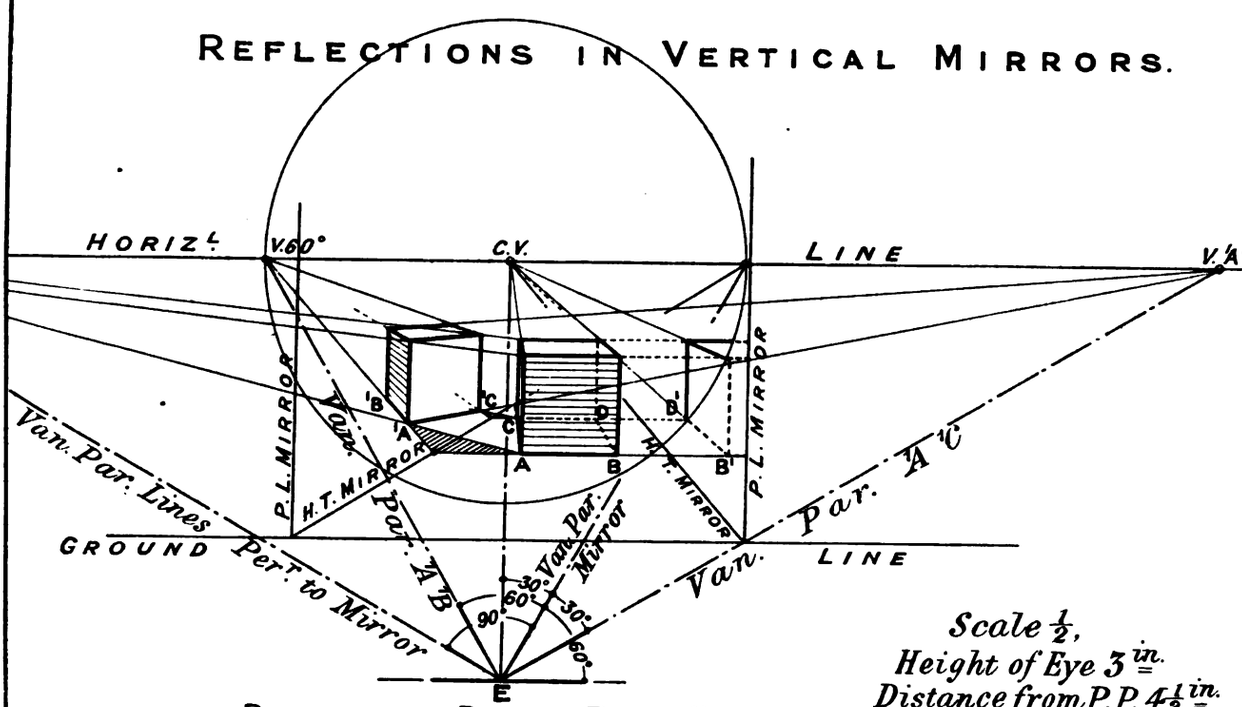
Again, the angle between CA and mirror is **30°**, so vanishing parallel ^{1}C ^{1}A also makes **30°** with vanishing parallel of the mirror—on the opposite side.

PLATE 27.



ORTHOGRAPHIC PROJNS

REFLECTIONS IN VERTICAL MIRRORS.



RADIAL OR PERS. PROJN

Scale $\frac{1}{2}$,
Height of Eye $3^{in.}_{=}$
Distance from P.P. $4\frac{1}{2}^{in.}_{=}$

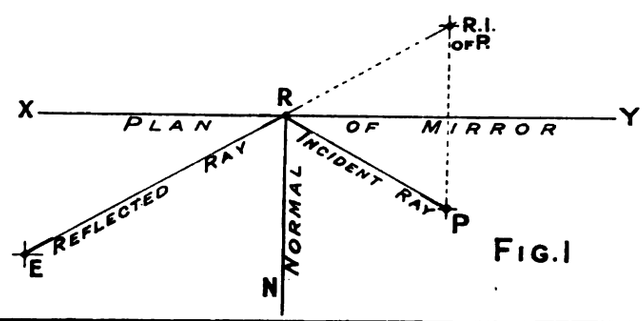


FIG. I

PLATE XXVIII.

REFLECTIONS IN A HORIZONTAL PLANE.

A cube and a pyramid rest on an ascending plane, the H.T. of which is $4\frac{3}{8}$" from the P.P. Its horizontal inclination is 40°.

One edge of the cube coincides with the H.T., its near end $1\frac{1}{2}$" to the right. Edge of cube $2\frac{3}{8}$". One edge of the square base of the pyramid also reaches the ground. Near end of it $\frac{7}{8}$" to the left. Length of base edge $2\frac{3}{8}$", altitude $3\frac{1}{4}$".

Assuming the G.P. in front of ascending plane to be a reflecting surface, show reflections of the two models so far as they may come within the Field of Vision.

Plan and elevation again show what has to be looked for as regards reflections; that is, the position of their reflected images; and seeing the reflecting surface is horizontal, we seek for a reversal of the models on this horizontal plane.

In the perspective diagram, then, having obtained the representation of the two models, we must proceed by obtaining the **V.L.** of a *descending* plane to represent the reflected image of the ascending one, together with the **V.P.'s** for diagonals of base; and also for that of lines perpendicular to the descending plane, and as under edge of each model touches the **G.P.**—the reflecting surface—we start here with the various lines required. Beginning with the cube, the reflected image of edge CD will vanish directly below **V.P.** for CD, and then as regards its length a perpendicular from D will give D'; or, as has been done in obtaining the perspective of the original figure overhead, a diagonal from **E.** will determine it; and so with base edges of pyramid. As for vertex of pyramid, it is best found by dropping a perpendicular from **V.** to meet **G.P.**—that is, to meet the **H.T.** of a vertical plane containing the axis—and continuing it as far below as **V.** is above.

It should be remembered here, as elsewhere, that equal distances on lines which are parallel to the **P.P.** measure equal distances in the perspective representation.

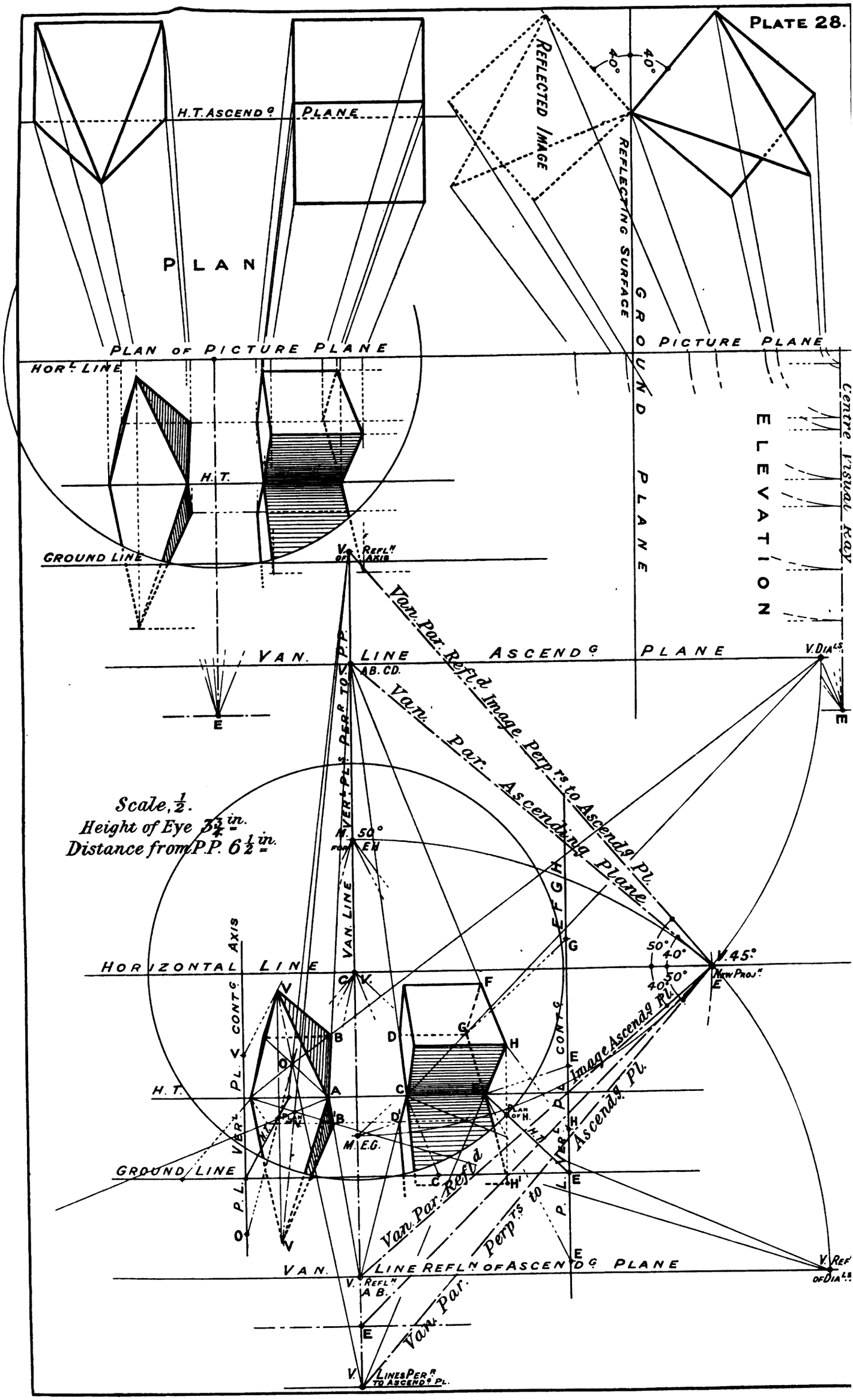

PLAN
H.T. ASCEND.G PLANE
REFLECTED IMAGE
REFLECTING SURFACE
40°
40°
GROUND PLANE
PICTURE PLANE
PLAN OF PICTURE PLANE
HOR.L LINE
H.T.
GROUND LINE
V. OF REFLN AXIS
ELEVATION
centre visual ray
VAN. LINE ASCEND.G PLANE
V.DIA.LS.
E
Van. Par. Refd. Image Perp.rs to Ascend.g Pl.
Van. Par. Ascending Plane
V. PP.
AB. CD.
VER.L PL.S PERP.R TO PP.
Scale, 1/2.
Height of Eye 3 1/4 in.
Distance from P.P. 6 1/2 in.
50°
VAN. LINE
HORIZONTAL LINE
V. CONT.G PL.
H.T.
GROUND LINE
P.L.
M.E.G.
PLAN OF H.
V. 45°
NEW PROJ.N
E
Image Ascend.g Pl.
Ascend.g Pl.
Van. Par. Refd
Perp.rs to P.P.
VAN. LINE REFLN OF ASCEND.G PLANE
V. REFLN AB.
V. REF OF DIA L.S
Van. Par.
V. LINES PERP.R TO ASCEND.G PL.

PLATE XXIX.

CUBE LYING ON AN OBLIQUE PLANE WITH REFLECTION IN A VERTICAL MIRROR.

A cube of $2\frac{1}{2}$" edge rests on a horizontal or ground-plane, which is $3\frac{1}{2}$" below the eye.

AB, the edge which rests on the ground, is inclined at 50° with the G.L. towards the right, A being $3\frac{1}{2}$" from the P.P., and directly in front of the spectator.

The diagonals of the vertical faces of the cube are horizontal and vertical.

Or we can describe its position by stating that the face ABCD lies in an oblique plane which is inclined **45°** to the horizon.

Required its reflection in a vertical mirror which meets the P.P. 7" to the right, the plane of the mirror being inclined 60° towards the left.

First find the intersection of the oblique plane containing ABCD and that of the mirror: which line will vanish to the intersection of their respective **V.L.'s.**

Then for image of **H.T.** oblique plane, its vanishing parallel makes an angle with vanishing parallel of the mirror equal to that made by the original **H.T.,** but on its opposite side.

Perpendiculars to the mirror (that is to **V.P. 30°** on right) drawn from B and A, determine B' and A', and the vertical diagonal through B' being drawn, can also be cut by perpendiculars to the mirror, both at centre and at top.

As regards edges of vertical faces, these faces being in planes perpendicular to the **H.T.,** we can obtain the required **V.L.** for them by a perpendicular to the vanishing parallel reflected image of **H.T.**

C'G' will now vanish to the horizontal of this **V.L.,** and for **V.P.** B'C', F'E' we set off at new projection of **E.** (for this **V.L.** of the reflection of the vertical faces) the angle of slope, **45°**, both above and below **H.L.**—above for B'C', below for A'E', etc.

Or we can bring DC across its plane till it meet the intersecting line, and carrying from this a line to **V.P.** for **H.T.** reflected image, we can check position of C'D'.

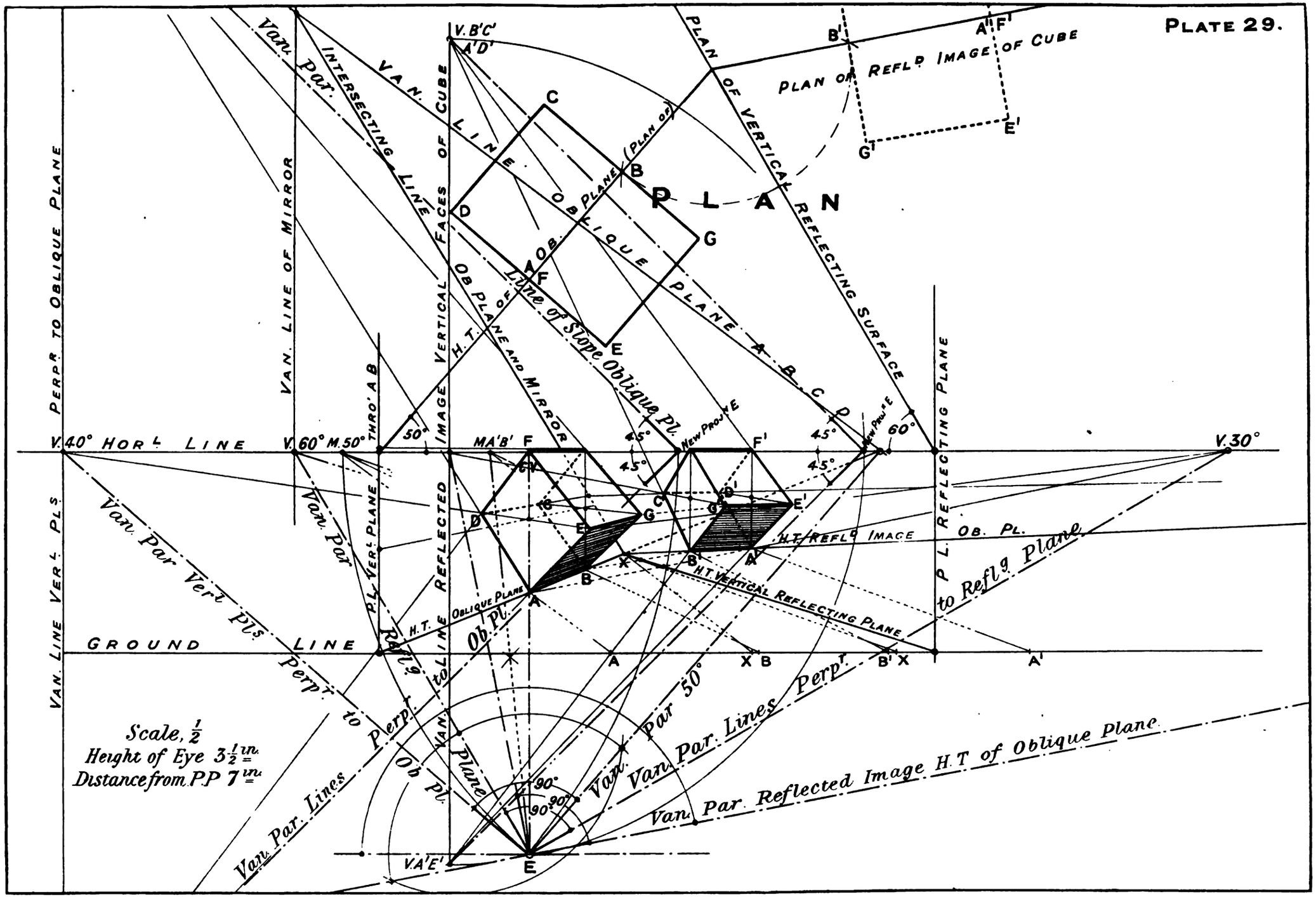


PLATE 29.
PLAN
PLAN OF REFL.D IMAGE OF CUBE
PLAN OF VERTICAL REFLECTING SURFACE
VAN. PAR. INTERSECTING LINE OF VERTICAL FACES OF CUBE
VAN. LINE OF MIRROR
PERP.R TO OBLIQUE PLANE
OB. PLANE (PLAN OF)
OBLIQUE PLANE A B C D
OB. PLANE AND MIRROR
H.T. OF Line of Slope Oblique Pl.
V. 40° HOR.L LINE
V. 60° M. 50° THRO' A B
50°
MA'B' F
NEW PROJ.N E
NEW PROJ.N F
45°
60°
V. 30°
VAN. LINE VER.L PLS.
VAN. PAR. PL. VER.L PLANE
IMAGE VERTICAL REFLECTED TO LINE
H.T. OBLIQUE PLANE
H.T. REFL.D IMAGE OB. PL.
H.T. VERTICAL REFLECTING PLANE
P.L. REFLECTING PLANE to Refl.g Plane
GROUND LINE
Refl.g Ob. Pl.
Van. Par. Ver.l Pls. Perp.r to Ob. Pl.
Van. Par. Lines Perp.r to Van. Plane
Van. Par. 50°
Van. Par. Lines Perp.r
Van. Par. Reflected Image H.T of Oblique Plane
V.A'E'
90°
Scale, ½
Height of Eye 3½ in.
Distance from P.P 7 in.

PLATE XXIX.

CUBE LYING ON AN OBLIQUE PLANE WITH REFLECTION IN A VERTICAL MIRROR.

A cube of $2\frac{1}{2}$" edge rests on a horizontal or ground-plane, which is $3\frac{1}{2}$" below the eye.

AB, the edge which rests on the ground, is inclined at 50° with the G.L. towards the right, A being $3\frac{1}{2}$" from the P.P., and directly in front of the spectator.

The diagonals of the vertical faces of the cube are horizontal and vertical.

Or we can describe its position by stating that the face ABCD lies in an oblique plane which is inclined **45°** to the horizon.

Required its reflection in a vertical mirror which meets the P.P. 7" to the right, the plane of the mirror being inclined 60° towards the left.

First find the intersection of the oblique plane containing ABCD and that of the mirror: which line will vanish to the intersection of their respective **V.L.'s**.

Then for image of **H.T.** oblique plane, its vanishing parallel makes an angle with vanishing parallel of the mirror equal to that made by the original **H.T.**, but on its opposite side.

Perpendiculars to the mirror (that is to **V.P. 30°** on right) drawn from B and A, determine B' and A', and the vertical diagonal through B' being drawn, can also be cut by perpendiculars to the mirror, both at centre and at top.

As regards edges of vertical faces, these faces being in planes perpendicular to the **H.T.**, we can obtain the required **V.L.** for them by a perpendicular to the vanishing parallel reflected image of **H.T.**

C'G' will now vanish to the horizontal of this **V.L.**, and for **V.P.** B'C', F'E' we set off at new projection of **E.** (for this **V.L.** of the reflection of the vertical faces) the angle of slope, **45°**, both above and below **H.L.**—above for B'C', below for A'E', etc.

Or we can bring DC across its plane till it meet the intersecting line, and carrying from this a line to **V.P.** for **H.T.** reflected image, we can check position of C'D'.

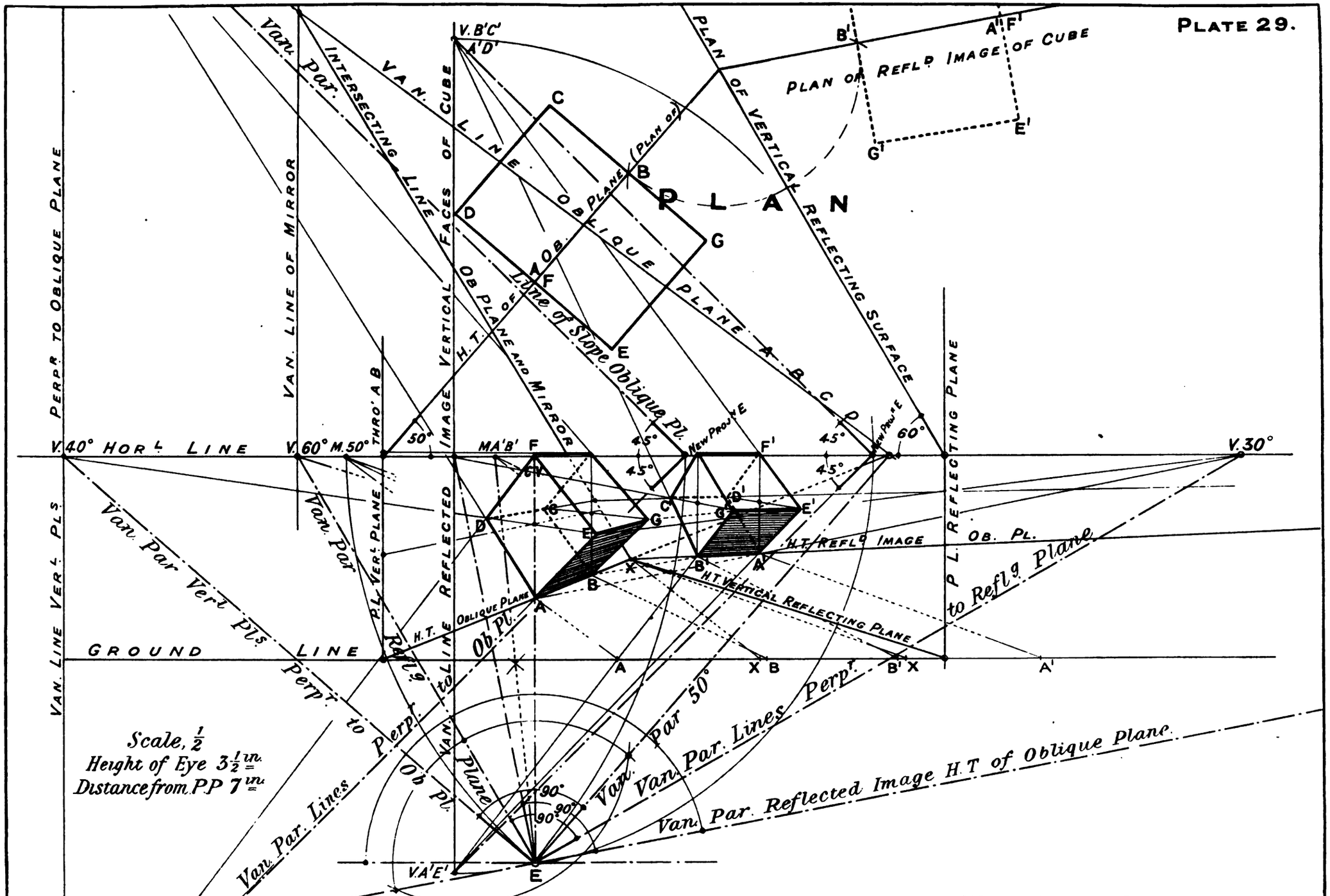

PLAN
PLAN OF REFL.D IMAGE OF CUBE
PLAN OF VERTICAL REFLECTING SURFACE
VAN. LINE OF MIRROR
INTERSECTING LINE OF VERTICAL FACES OF CUBE
OB. PLANE AND MIRROR
LINE OF SLOPE OBLIQUE PL.
OBLIQUE PLANE A B C D
PERP.R TO OBLIQUE PLANE
V. 40° HOR.L LINE
V. 60° M. 50°
V. 30°
GROUND LINE
VAN. LINE VER.L PLS.
P.L. VER.L PLANE
REFLECTED IMAGE
P.L. REFLECTING PLANE
H.T. REFL.G IMAGE OB. PL.
H.T. VERTICAL REFLECTING PLANE
H.T. OBLIQUE PLANE
Ob. Pl.
Refl.g to Line
Van. Par. Plane
Van. Par. 50°
Van. Par. Lines Perp.r to Ob. Pl.
Van. Par. Lines Perp.r
Van. Par. Reflected Image H.T of Oblique Plane
Van. Par. Ver.l Pls. Perp.r to Refl.g Plane
Scale, 1/2
Height of Eye 3 1/2 in.
Distance from PP 7 in.

PLATE XXX.

Given corner of a room in perspective, with framed picture suspended on left-hand wall, to obtain its reflection in mirror on right-hand wall, the walls being at right angles.

Horizontal lines on both walls being continued to meet will give **V.P.'s**, and hence the **H.L.**

And now to continue horizontal edge AB as far beyond intersecting line of walls as AB is to left.

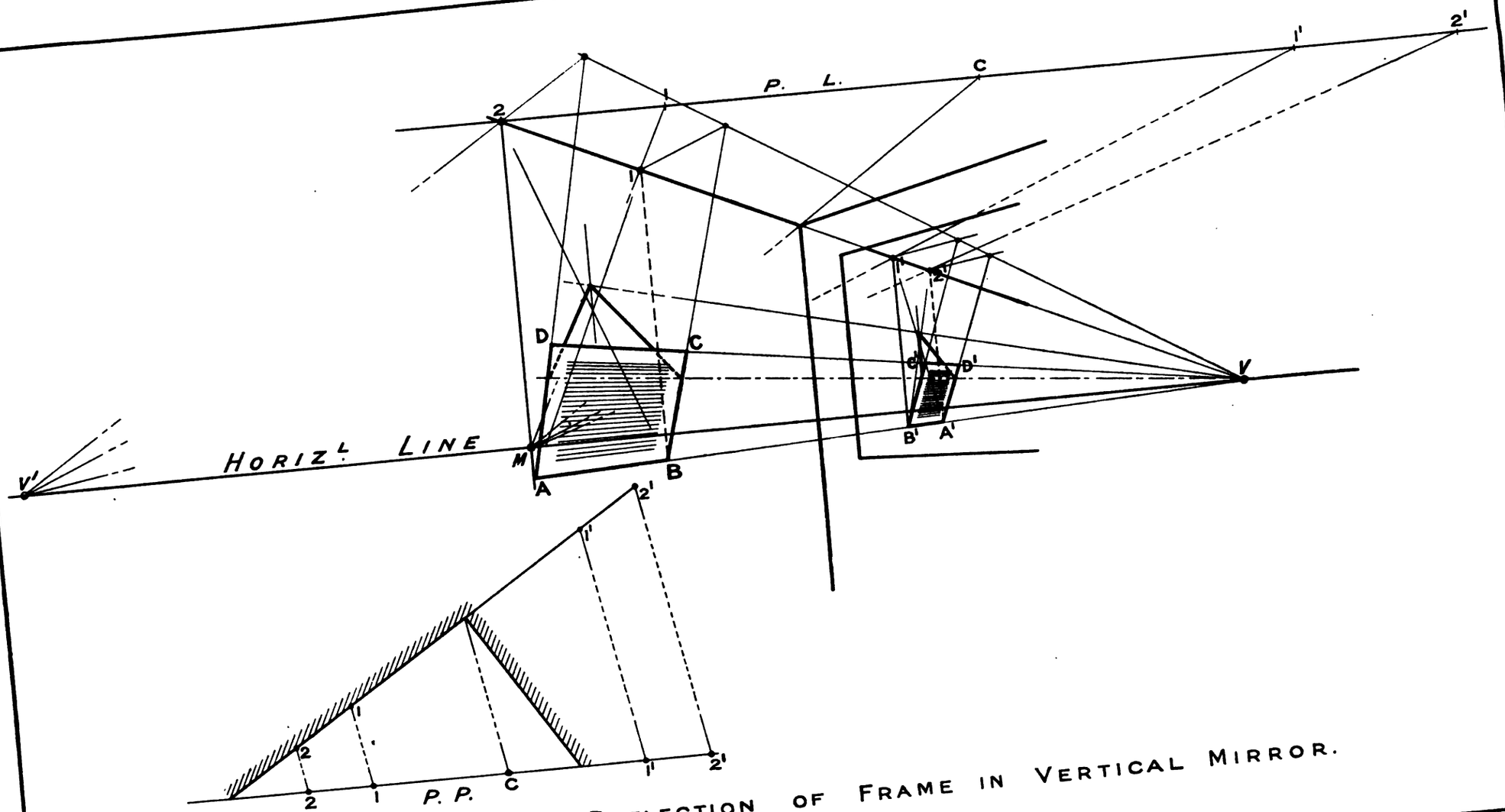
Erect perpendiculars to meet junction with ceiling at 2 and 1, and to reverse distances 1 and 2 from C to the opposite side, select *any* suitable line across ceiling parallel to the **H.L.** for a **P.L.** Use also *any* point **M.** on the **H.L.** to measure distances from C to the left, and to set off equal distances from C to the right. Of course it will be understood that these are not absolute distances—in fact, there are no absolute dimensions known, or required in this diagram.

In order to see that for this purpose any **M.P.** may be selected, we may consult the sketch plan showing a **P.P.** and any two lines representing the two walls at right angles.

A length on the one, as 1, 2, is required to be reproduced in a continuation of this wall beyond the right-hand one.

If we draw any parallels through C 1 and 2 to meet the **P.P.** and set off equal distances 1^1 and 2^1 on other side of C, parallels will cut wall line produced at points required. As regards the tilt forward of the frame, we may either obtain **V.P.** for edges AD and BC which lies in the **V.L.** through V^1, or producing them to meet the plane of the ceiling, as shown, reproduce similar triangles as their reflected images. Determine middle vertical line containing support for cord by diagonal intersections.

PLATE 30.



REFLECTION OF FRAME IN VERTICAL MIRROR.

PLATE XXXI.

A gateway with buttressed ends is shown in plan and elevation, it overlooks still water, the water-line being 9″ below the G.L.

The vertical front face of gateway meets the P.P. 5′ to the right of the centre, and vanishes 45° to the left. The nearest corner is 4′ from the G.L., measured along the H.T. of the wall.

Behind and running parallel to the gateway is a bank which slopes upward at 30°. Bottom edge of bank 4′ back, height of bank 2′.

Place all this in perspective, and project shadows when the sun is behind the spectator's left, the rays being in vertical planes which make 35° with the P.P. towards the right, altitude 30°.

Show also reflections in the still water in front. Height of eye above G.P. $5\frac{1}{4}$′. Distance from P.P. $11\frac{1}{2}$′.

Having obtained the **V.** and **M.P.'s** required, set off **P.L.** of front face of gateway, and measure heights on this to meet the various widths which again can be found in the **H.T.** of the wall. Notice that sloping surfaces of wall make **30°** with the horizon. To obtain lines of the bank assume a section going 4′ back through D, perpendicular to the face of the gateway, then rising at **30°**. Its height being measured by a horizontal line 2′ high.

To find shadows, a vertical through **V.P.S.R.** to cut **H.L.** is **V.P.** for shadows on **G.P.** of vertical lines; and when continued on sloping bank, the same vertical continuation to cut **V.L.** of bank will give **V.P.** required.

Shadow of FA is partly caught on sloping top EDF. Slope, therefore, is in light, and so edge DE, together with sloping line above **E.**, will throw shadows on bank until shadow of FA interrupts, being now continued on the bank. This continuation best found by starting from bottom of edge AF, where it reaches ground. Upper horizontal surface of bank receives shadow also.

In working out reflections, the water-line must be drawn in order to limit what may be seen.

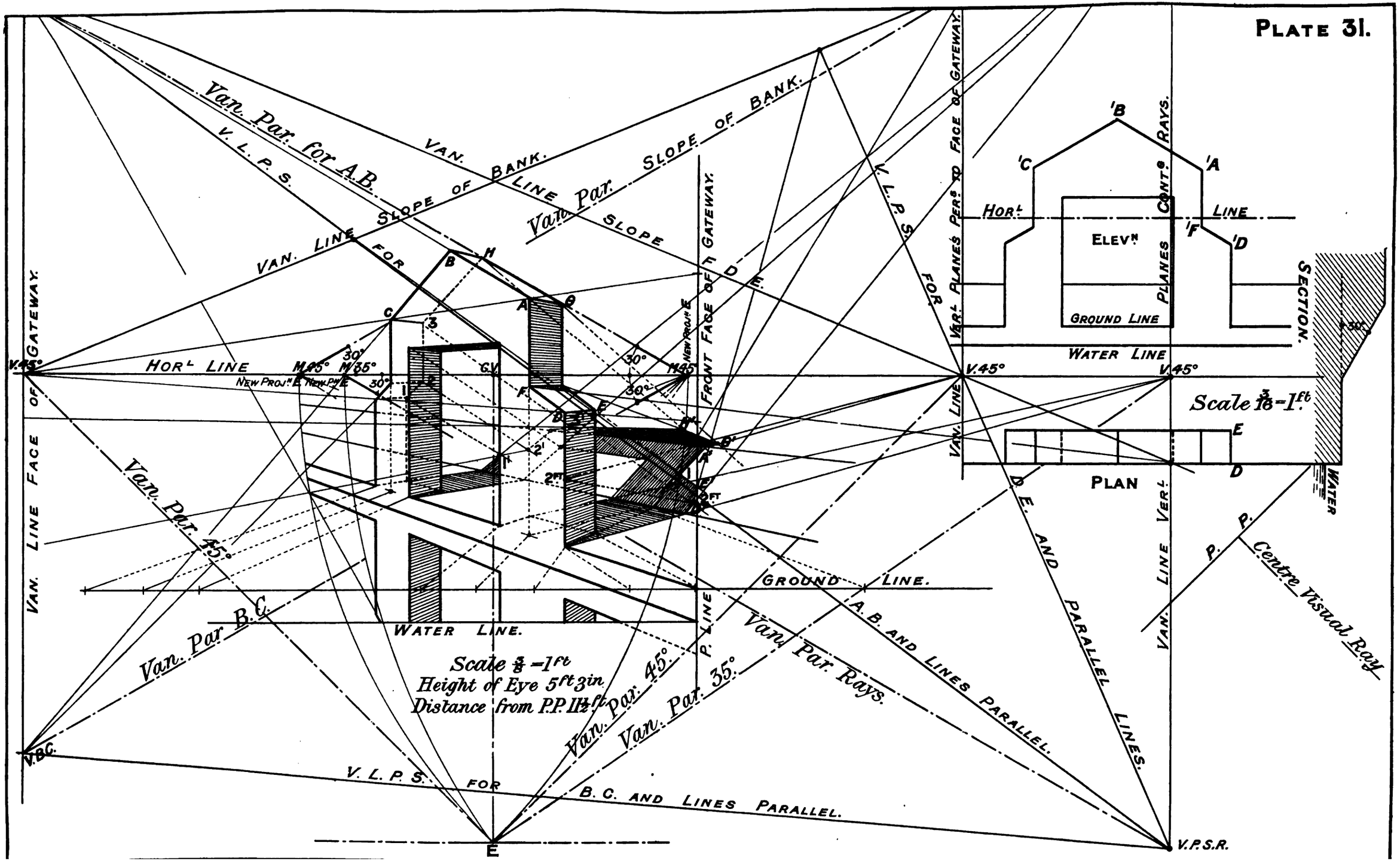


PLATE 31.
Van. Par. for A.B.
V. L. P. S.
VAN. SLOPE OF BANK.
Van. Par. SLOPE OF BANK.
VAN. LINE FOR
Van. Par. SLOPE
FRONT FACE OF GATEWAY.
V. L. P. S. FOR
VAN. LINE FACE OF GATEWAY.
HOR.L LINE
V.45°
M.45°
M.35°
30°
NEW PROJ.N E
NEW PROJ.N E
C.V.
A
B
C
D
E
F
H
1
2
3
1'
2'
2nd
3rd
Van. Par. 45°
Van. Par B.C.
GROUND LINE.
WATER LINE.
P. LINE
Van. Par. 45°
Van. Par. 35°
Van. Par. Rays.
A.B. AND LINES PARALLEL.
D.E. AND PARALLEL LINES.
Scale 3/8 - 1ft
Height of Eye 5ft 3in.
Distance from P.P. 11½ ft
V.B.C.
V. L. P. S. FOR B.C. AND LINES PARALLEL.
E
V.P.S.R.
'B
'C
'A
'F
'D
HOR.L LINE
ELEVN
CONT.D PLANES
RAYS.
VAR.L PLANES PERS.D TO FACE OF GATEWAY.
GROUND LINE
WATER LINE
SECTION.
Scale 3/16 - 1ft.
PLAN
VAN. LINE VER.L
P.
P.
Centre Visual Ray.
WATER

PLATE XXXII.

Projections of a small foot-bridge over a stream are shown, to scale of $\frac{1}{24}$.

It is required to draw a perspective representation when the H.T. of vertical face of near wall meets the G.L. 4′ to the left. The walls vanish 45° towards the right. Nearest corner A 4′ along this H.T.

The eye is $4\frac{1}{2}$′ above the G.P. and $7\frac{1}{2}$′ from the P.P. The sun being in the plane of the picture on the spectator's left, at an altitude of 45°.

Show shadows and reflections which may be visible.

Obtain **V.P.'s** on right and left **45°** for sides of bridge and steps, and also for horizontal lines of banks of the stream.

Find also **V.L.'s** for slope of bank and also for crest of wall.

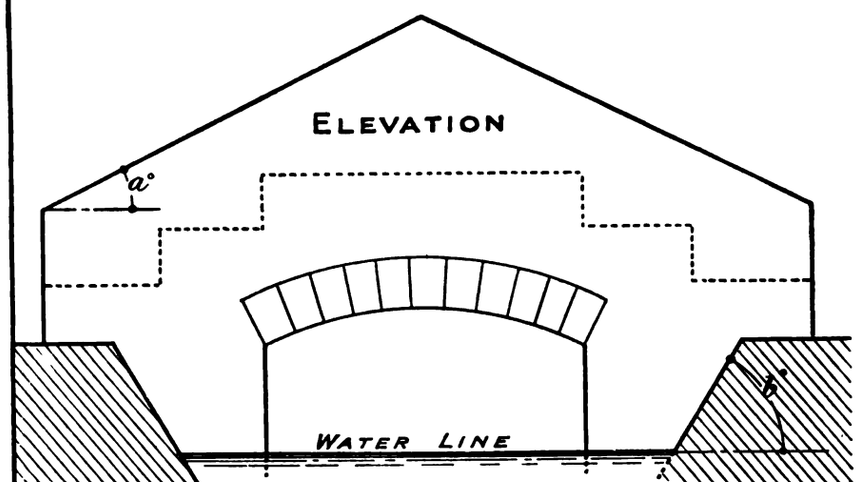
As for the shadows, begin with that of the vertical line through A, and then—on the other side of the stream—find shadow of corner B from that of the vertical through B. A parallel to the rays—that is, a line at **45°** through **V.P.** for slope of wall—will give, on the **H.L.**, the **V.P.** for shadow of slope coming towards edge of bank; and this again continued over the bank by a prolongation of this **V.L.** plane of shade to **V.L.** slope of bank, giving the required **V.P.** A ray through apex of wall will determine length of last line; and still there will be room for a short bit of shadow of near line of wall-slope. The **V.P.** for this is beyond the limits of the paper, but it is found by a **45°** line through **V.P.** of front slope to meet **V.L.** bank slope.

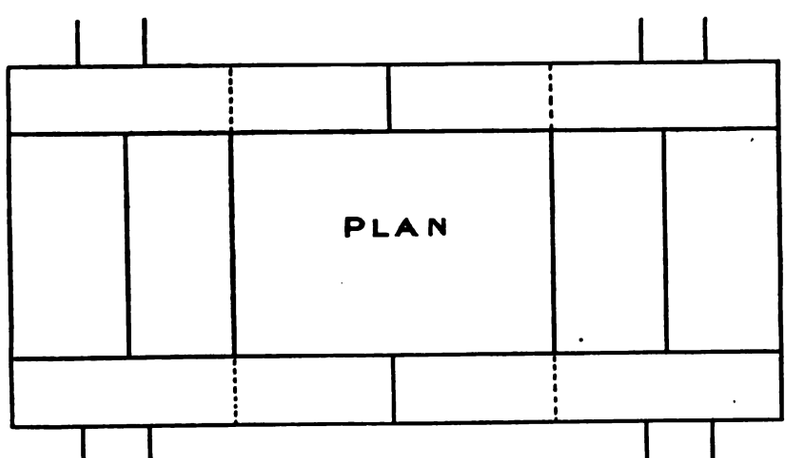
As for shadows on steps, near upright edge C will throw a shadow parallel to **P.P.**, and for shadow of sloping line either a line parallel to rays through its **V.P.** overhead, to meet **H.L.**, may be used as **V.P.** for shadows on horizontal surfaces, and the same line produced to meet **V.L.** upright surfaces will give **V.P.** for rising shadows.

These shadows may be obtained otherwise: starting with shadow of corner C, produce slope to meet horizontal surface of step—that is, find its **H.T.** on this surface—shadow of slope will then converge to this point; and having reached junction with upright surface of second step, produce this vertical plane to meet slope at D. Shadow will then be deviated to D. The second horizontal surface and upright of third may be treated in a like manner.

PLATE 32.

BRIDGE OVER A STREAM.





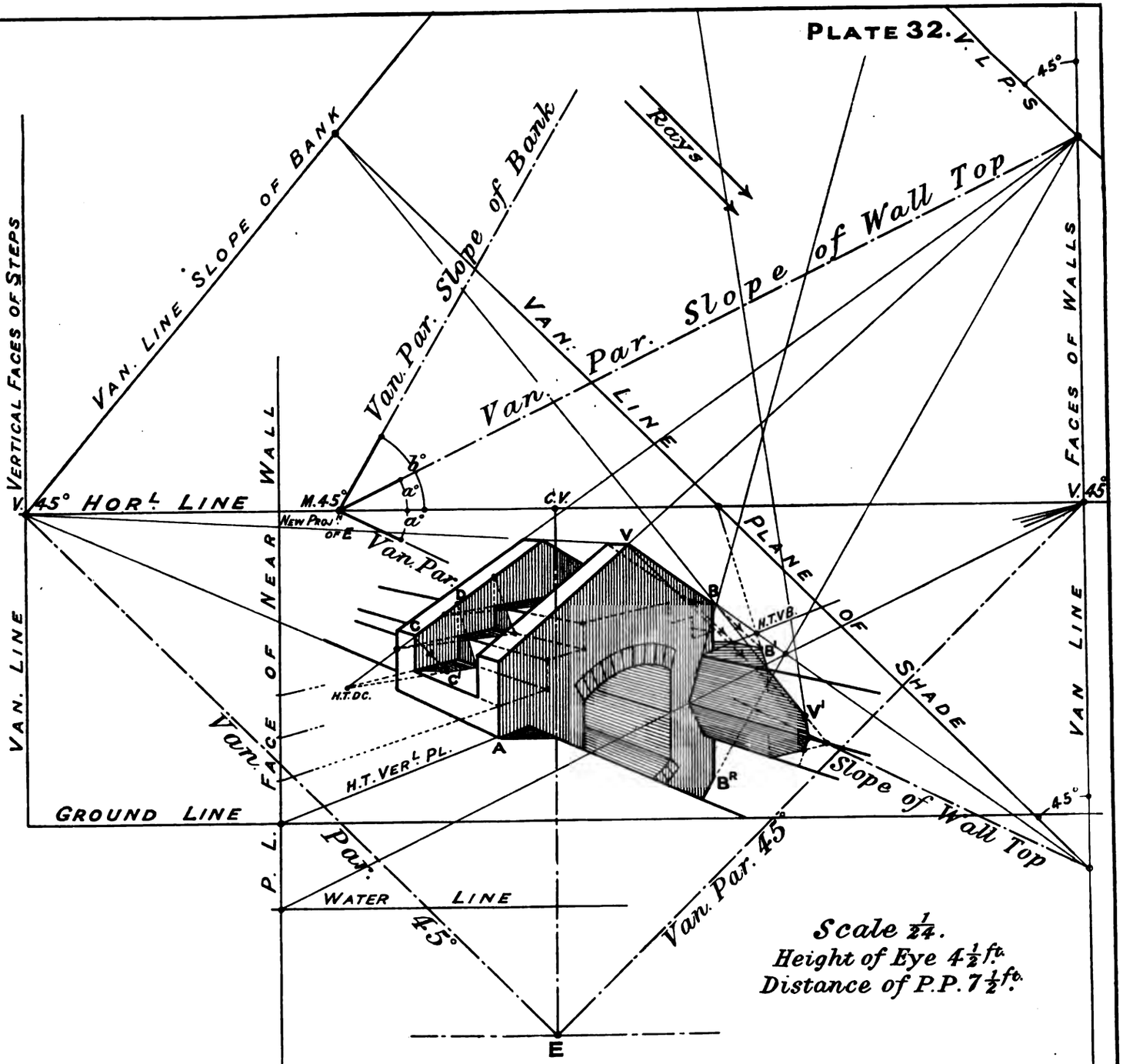


PLATE XXXIII.

MISCELLANEOUS PROBLEMS.

Fig. 1.—When a line is parallel to the **P.P.**—horizontal or otherwise—equal distances in it will be represented in perspective by equal distances.

In the similar triangles $2E3$ and 2^1E3^1, $2^1 3^1$ is to 2 3 as $E2^1$ is to $E2$; and again in triangles $2E1$ and 2^1E1^1, $2^1 1^1$ is to 2 1 as $E2^1$ is to $E2$, and hence $2^1 3^1 : 2^1 1^1 :: 2\ 3 : 2\ 1$. 2 3 and 2 1 are equal, therefore $2^1 3^1$ and $2^1 1^1$ are also equal; and so with the other equal distances along the line AB.

Fig. 2.—Problem.—To draw a line to a V.P. assumed to be inaccessible. —Suppose it is required to draw from P a line to a **V.P.**, the vanishing parallel of which is given. Set off any triangle having P for one angular point, another in the **H.L.**, and a third in the vanishing parallel. Here P is joined to **C.V.**, and also to **E.**; then by drawing a similar triangle, similarly placed, we obtain another point, P^1, in the line required. This property of similar triangles may be adopted also in determining the vanishing parallel (and hence its inclination to the **G.L.**) when a line is given in perspective with its **V.P.** beyond reach.

Fig. 3.—It has been already stated that the **P.P.** need not be vertical in all cases.

Examples may arise where we have figures lying on a **G.P.** with the **P.P.** tilted over, as is shown in the side elevation (Fig. 3).

In that case we treat the **G.P.** as an ascending plane, measuring its angle of tilt by a perpendicular to the **P.P.**, as angle $a°$.

Fig. 4.—Problem.—To find a point 40′ in the right of the centre, 80′′ within the picture, and 30′ high.

Since, as shown in Fig. 1, distances on lines parallel to the **P.P.** measure proportionate distances in perspective; and because it may be inconvenient or impossible to set off on the **G.L.** the scale-distance required on either side of the centre or within the picture, we can adopt a method of measuring by lines which are parallel to the **P.P.**, but sufficiently far removed to render this measuring possible. As here required, first, a line on the **G.P.** perpendicular to the **G.L.**, which will be 40′ to the right: set off on right of centre of **G.L.** some convenient submultiple of 40, as 10; and by joining to **C.V.** cut off on any suitable line, as A—parallel to **G.L.**—a distance which will represent 10′; and this being repeated along line A four times, and again joined to **C.V.**, will give a line 40′ to the right.

By joining centre of **G.L.** to **P.D.** we obtain a point P in this line 40′ within; and in this special case—the distance within being twice 40—by bringing a parallel through P to meet middle line at C, and again joining to **P.D.**, we obtain the 80′ distance. In general, however, we require to draw and measure another line as B parallel to **G.L.** with some submultiple of the distance required beyond what the centre line may give us.

As example, suppose the required distance within to be 85′, we would then, on this line B, and having found a new scale of distances, say of 10′, set off 45 to left of centre, and join to **P.D.** As regards height, the **H.L.** being here 5′ above **G.P.** will give a unit of measurement for the required height; repeated six times will give the required 30′.

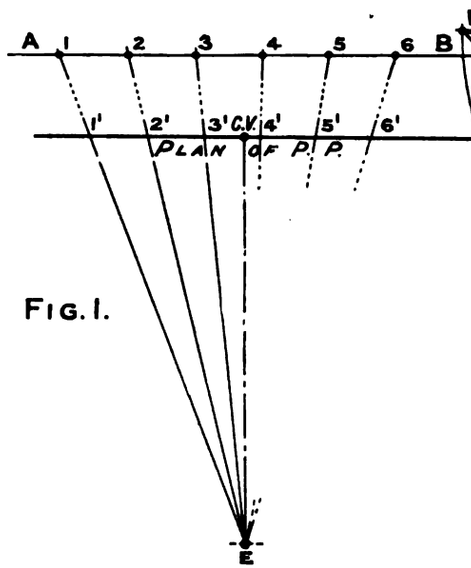


FIG. 1.

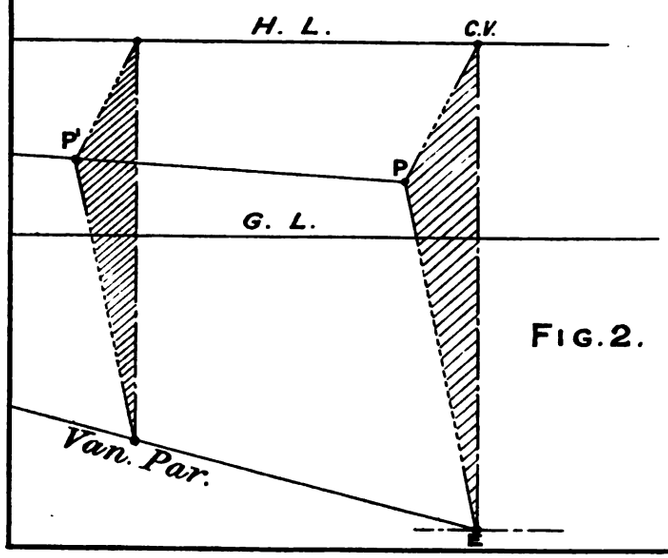
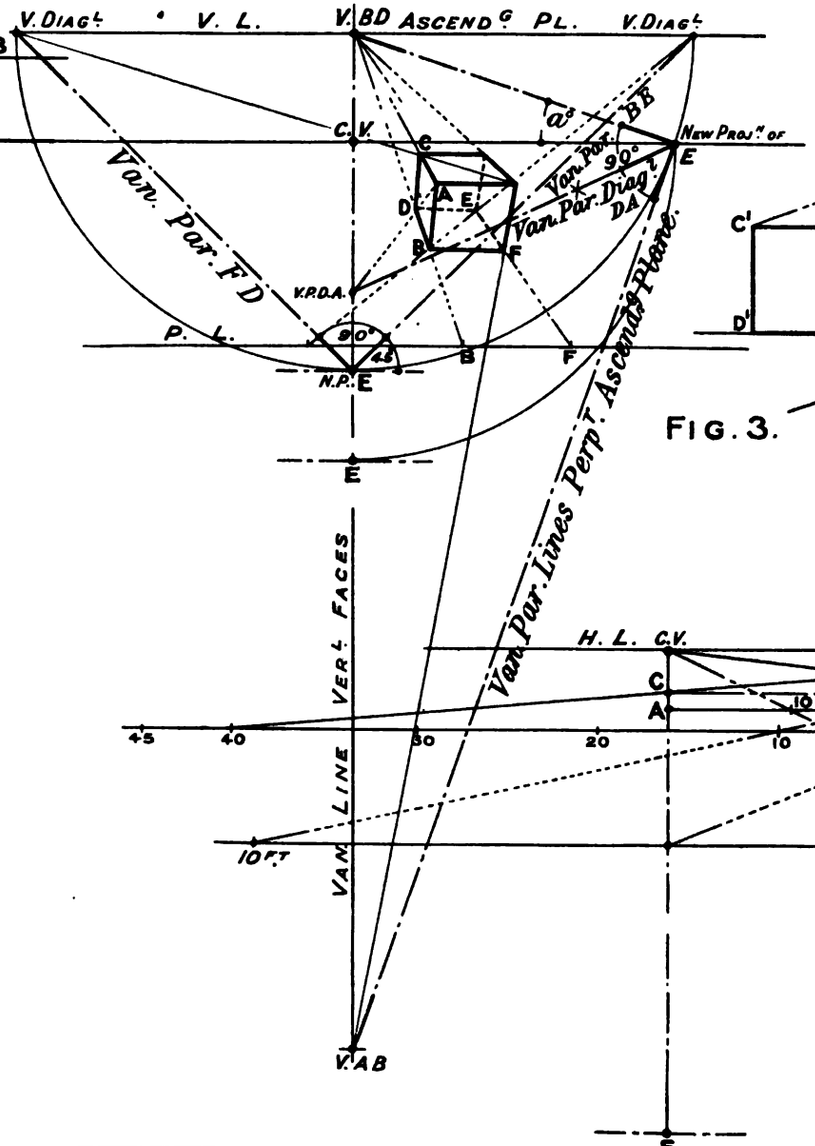


FIG. 2.



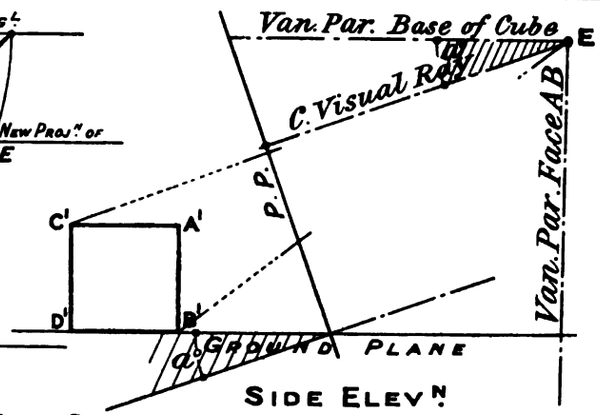


FIG. 3.

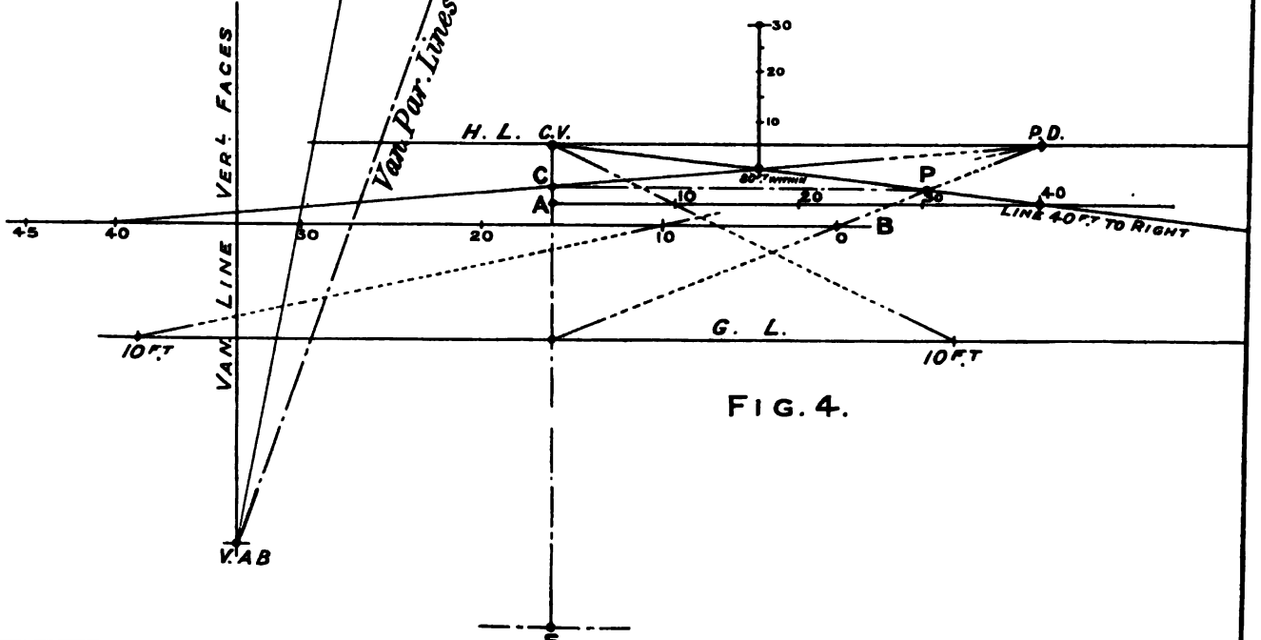


FIG. 4.

PLATE XXXIV.

PROBLEMS FROM EXAMINATION PAPER (April, 1901).

Question 1.—Diagram shows in perspective a tower, square on plan, standing on the ground-plane, its nearest edge marked AB, and, raised above the ground, a horizontal plane on which is a point A^{I}. The horizon is at H.L. and C.V. E. is the distance of the eye from the picture. At point A erect a vertical line perspectively equal to AB.

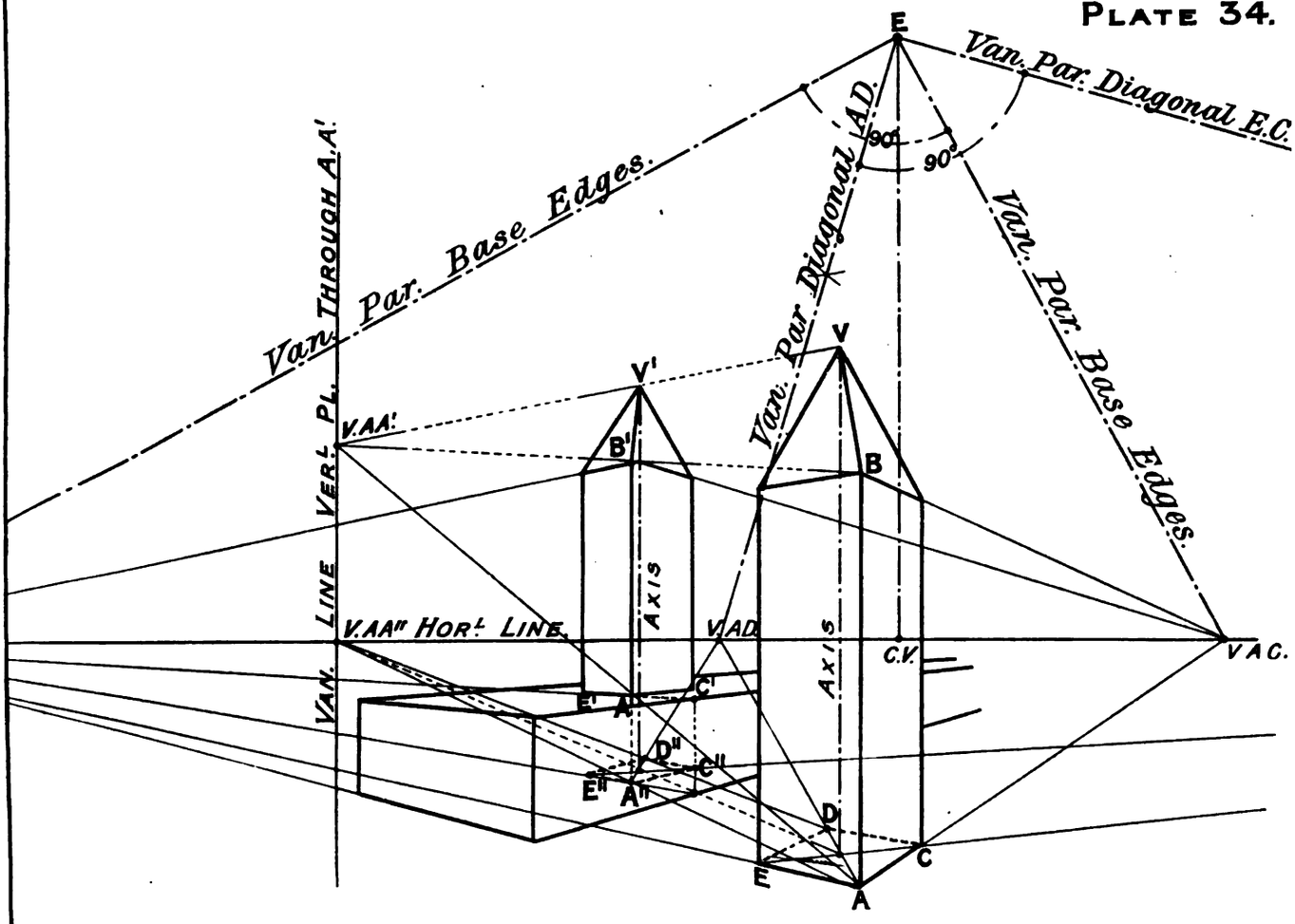
This to be the nearest edge of another similar tower standing on the raised plane, and having the same dimensions, and its sides inclined to the picture at the same angles as the tower shown.

In this example the position of **E.** is not necessary for the working out of the problem. In order, however, to check correctness of drawing—which is here reduced from the given diagram—the **V.P.'s** have been found. The first thing requisite is to find the **V.L.** for the vertical plane containing A and A^{I}, and for this we must find the plan of A^{I} on the **G.P.**; a cross-section of the raised plane will give this plan A^{II}; AA^{II} joined will be the **H.T.** of the required plane, and this produced to **H.L.** will determine its **V.L.** **V.P.** for AA^{I} is next found in this **V.L.**, which enables us to get $A^{I}B^{I}$. For base of tower this had better be reproduced at A^{II} on the **G.P.**, and then carried aloft. This square, $A^{II}D^{II}C^{II}E^{II}$, is most *exactly* obtained by first transferring diagonal AD and centre. As regards vertex, VV^{I} will be parallel to AA^{I}.

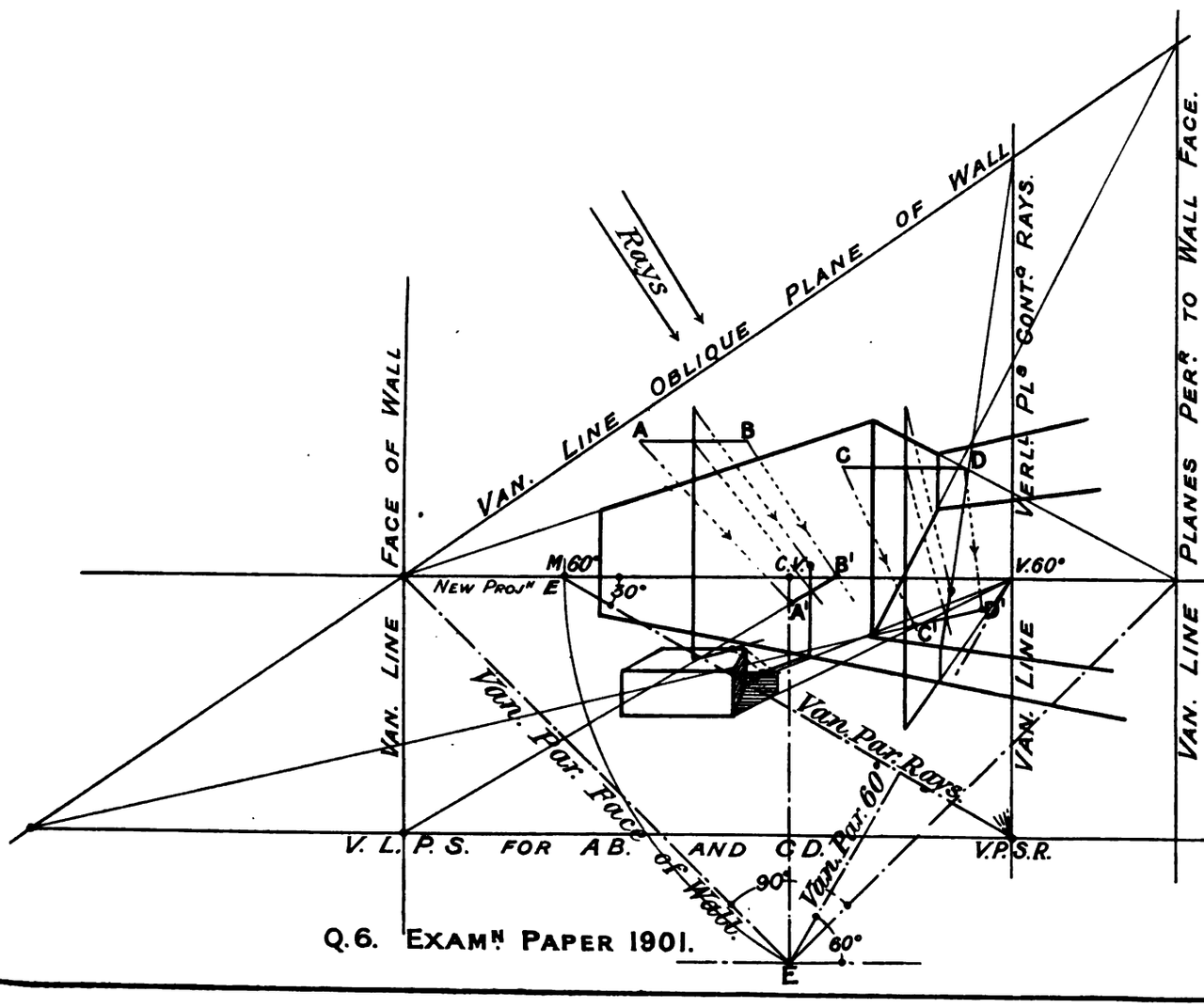
Question 6.—The diagram represents in perspective a wall composed of vertical and oblique planes and two crosses, the one on the right standing on the G.P., the other raised on a rectangular block, which also stands on the G.P.

The position of the horizon, the centre of vision and the distance of the eye from the picture are given. Draw the shadows cast by these objects, the sun being behind the spectator in a vertical plane receding to the right and inclined at 60° with the P.P., the altitude being 30°.

There is nothing in the working out of this calling for special remark. Notice, however, that, having carried shadow of left-hand cross over the top of the block, we continue down a ray of light till we reach shadow of back edge of block, and continue from there shadow of cross on the **G.P.** As for shadows of horizontal arms of crosses, being lines parallel to the **P.P.**, we find **V.P.'s** for their shadows by drawing a parallel to them through **V.P.S.R.** to to meet the **V.L.'s** of the planes receiving their shadows.



Q.1. EXAMN. PAPER 1901.



Q.6. EXAMN. PAPER 1901.

PLATE XXXV.

PROBLEMS FROM EXAMINATION PAPER (April, 1901).

Question 4.—Diagram shows in perspective the main lines of a barge (rectangular in plan and cross-section) lying on an oblique plane, with one edge on the ground-plane, and a point A on its upper surface. The position of the horizon, the centre of vision, and the ground-line are given. The distance of the eye from the picture is 11′. At point A erect a mast 8′ long, perpendicular to the upper surface of the barge.

Finding the **V.P.** for the horizontal of the bank, we can then obtain the **V.P.** for planes perpendicular to it.. And since the barge is rectangular in plan, its end lines (ignoring the far edge of the oblique plane) will give the **V.P.** for the slope of the bank.

And again, having laid down its vanishing parallel, a perpendicular to it will give **V.P.** for the mast. Next, by tracing a line across the upper surface of the barge through A and down its side, by using the appropriate **V.P.'s**, we reach the **H.T.** of the oblique plane, and from there draw across the **G.P.** the **H.T.** of the vertical plane containing the mast, and so reach the **P.P.** and the **P.L.**, on which—after finding its **M.P.**—we can now measure the 8′ mast.

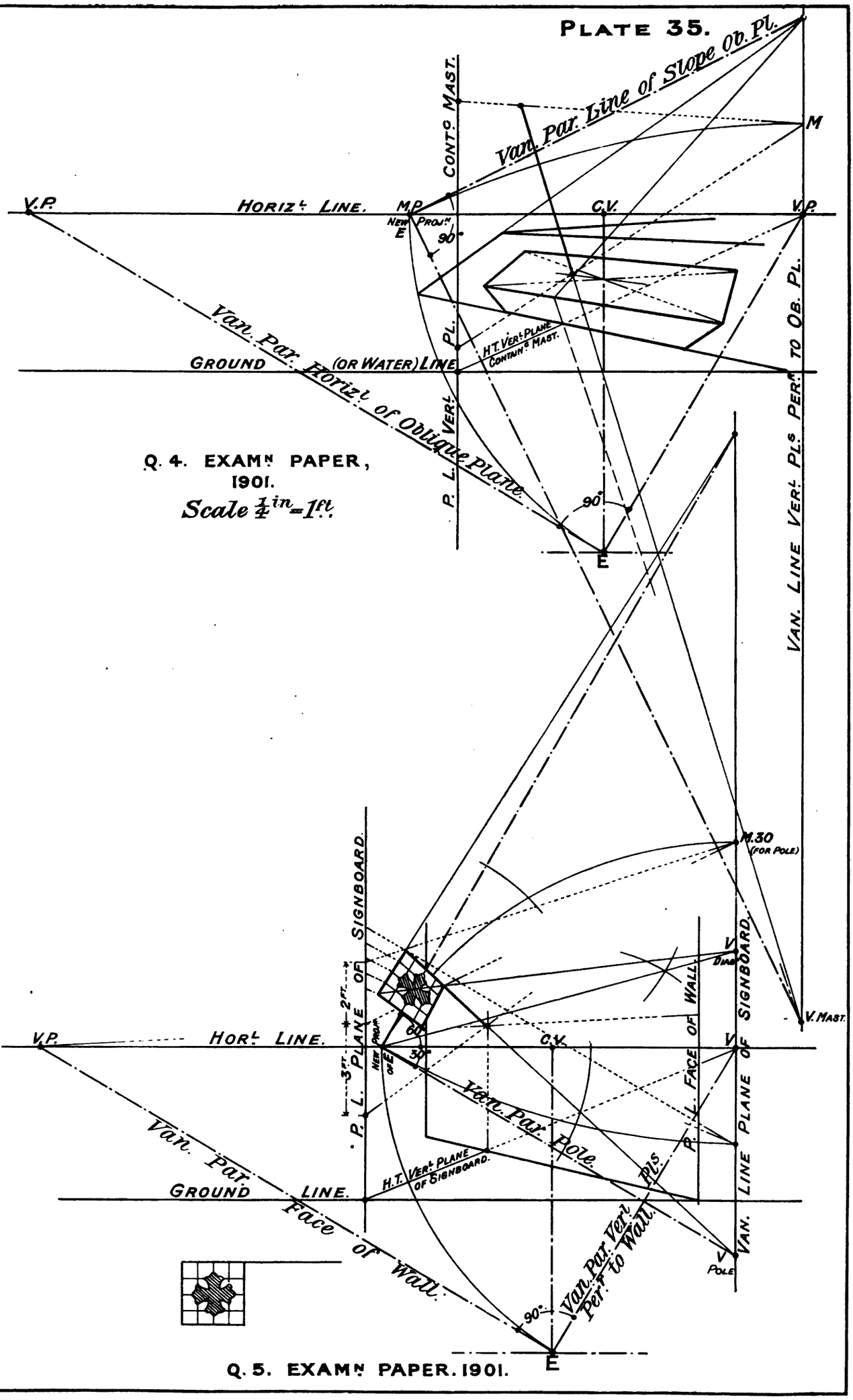
Question 5.—Diagram shows in perspective a vertical wall, and at its foot a point A on the ground-plane. C.V. E is the distance of the picture from the eye, which is 5′ above the ground-line G.L.

From a point on the wall 6′ vertically over A projects upwards a line 5′ long to represent a post lying in a vertical plane perpendicular to the plane of the wall, the line making an angle of 30° with the ground-plane. The upper end of the post is to bear a sign-board 2′ square, with the device shown on the small diagram.

In this example it has been found necessary to change the position of A from that given in the Examination Paper, in order to be able to carry out the requirements as to angle of post.

The working out is similar to that involved in Question 4: finding the **V.L.** and **P.L.** of a vertical plane through A, and in this the **V.P.'s** for **30°** below and **60°** above the horizon, with **M.P.'s** to correspond.

PLATE 35.



Q. 4. EXAMN PAPER, 1901.

Scale $\frac{1}{4}$ in = 1 ft.

Q. 5. EXAMN PAPER. 1901.

PLATE XXXVI.

PROBLEMS FROM EXAMINATION PAPER (April, 1901).

Question 12.—Diagram gives in perspective a vertical surface standing on a horizontal plane.

AB is a vertical line erected on the same plane, and casts the shadow shown by the dotted line, as does the line DF, which is perpendicular to the vertical plane. The horizon, the centre of vision, and the distance of the eye from the picture are given. Find the source of the artificial light, and draw the shadow of the line HI, which is parallel to DF.

In order to obtain the position of the source of light, the rays through B and F will intersect and give **L.** Its projection on the **G.P.**, or its plan, is, however, requisite in order to define exactly its position. AB is sufficient for this because, being vertical, its shadow will be in the **H.T.** of a vertical plane containing **L.**, and so a vertical from **L.** will cut this and show its plan. The plan of DF and its shadow may also be utilized for defining position of **L.** And to obtain shadow of HI we find its plan, and a line through **L.**, and plan of I will be **H.T.** of a vertical plane containing shadow of I, which is then obtained in the intersection line of this vertical plane with the surface receiving the shadow.

Question 10.—Diagram gives a perspective view of a road descending from the ground-plane, H.L. being the horizon. At point A, which is on the ground-plane, erect a vertical line to represent a man, equal in height to the nearest edge of the cottage door, which is assumed to be 6′. Find also the height of the tree standing at point B.

Obtain the **V.L.** for the vertical plane of cottage front, and this will contain **V.P.** for the bottom line of the wall, which is in the descending plane. From this **V.P.** bring the 6′ height of door up to a vertical at the junction of the descending and horizontal planes of the road. And from the **V.P.** for the horizontals in the same plane bring it forward into line with A, to which it can be thus transferred.

A 6′ height on the tree can be obtained in a similar manner, and this forms a scale for measuring its height (about 24′).

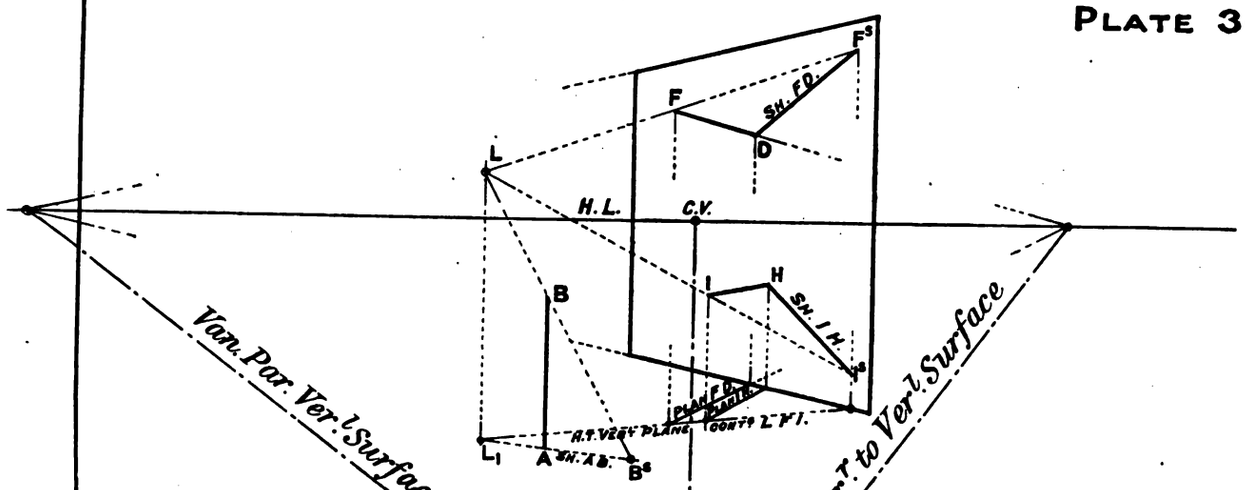
Question 9.—Diagram is a perspective view of a rectangular pier supporting the remains of a semicircular arch, the springing-line of the arch being at level Sp. On the right is shown the trace on the ground of the other pier. The horizon, the centre of vision, and the distance of the picture from the eye are given.

Restore the whole archway.

The **V.P.'s** being found, the springing-line of the arch is carried over to meet the near pier, and to obtain centre of arch, in front, draw diagonals. An upright will then give the centre on the springing-line.

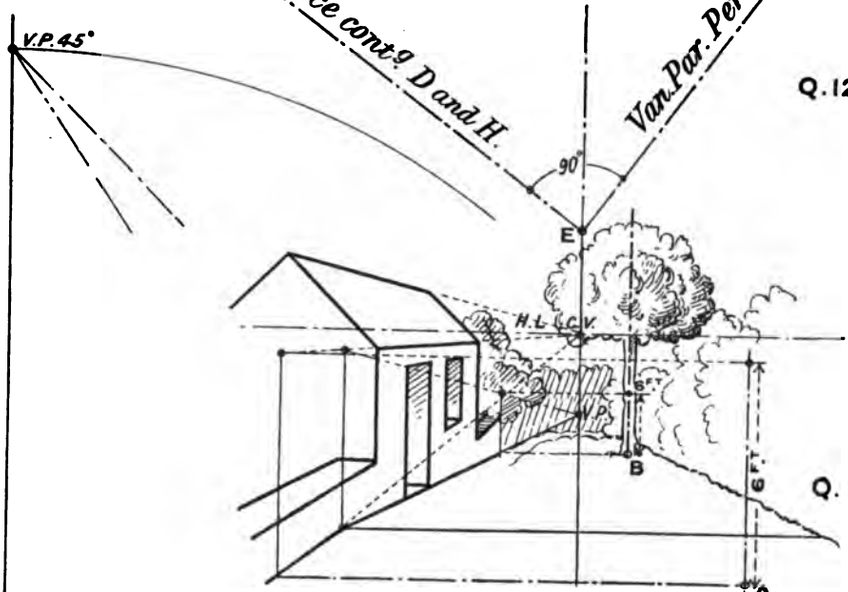
Find next **V.P.** for **45°** (either will do) in the **V.L.** of the vertical face of the archway, and a line from the centre to this will determine top corner of rectangle containing the semi-ellipse. The rest of the construction is obvious.

PLATE 36.



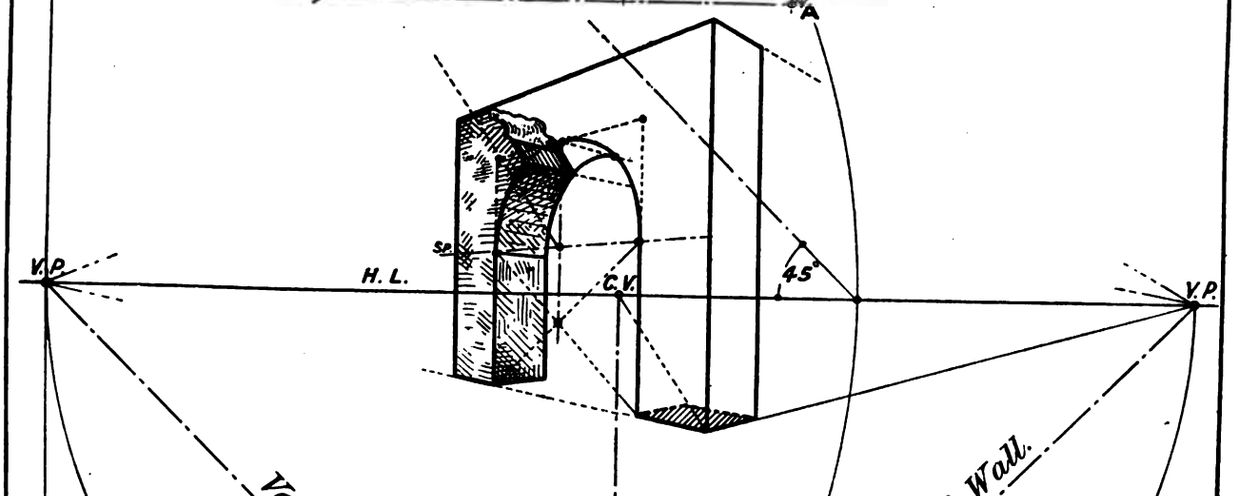
Van. Par. Ver.l Surface cont.g D and H.

Van. Par. Per.r to Ver.l Surface



Q. 12. EXAMN PAPER.
1901.

Q. 10. EXAMN PAPER.
1901.



Q. 9. EXAMN PAPER.
1901.

Van. Par. Face of Wall

Van. Par. Thickness of Wall.

E

THE END.

PRINTED BY WILLIAM CLOWES AND SONS, LIMITED, LONDON AND BECCLES.

10 [illegible]52

39
39a

89080452139

B89080452139A